Differential
Equations
& Applications
Volume 7, Number 1 (2015), 1–13
doi:10.7153/dea-07-01

ASYMPTOTIC PROPERTIES OF SOLUTIONS TO A NONLINEAR SYSTEM OF NEUTRAL DIFFERENTIAL EQUATIONS

JULIO G. DIX, EVA ŠPÁNIKOVÁ AND HELENA ŠAMAJOVÁ

Abstract. In this article we study the behavior of solutions to the system of delay differential equations

$$\begin{aligned}
\left[y_1(t)+a(t)y_1(g(t))\right]' &= p_1(t)f_1\left(y_2(h_2(t))\right)\\
y_2'(t) &= p_2(t)f_2\left(y_3(h_3(t))\right)\\
&\cdots\\
y_{n-1}'(t) &= p_{n-1}(t)f_{n-1}\left(y_n(h_n(t))\right)\\
y_n'(t) &= f_n\left(t,y_1(h_1(t))\right),
\end{aligned}$$

where the coefficients p_i may have zeros, and the components of the solution may change signs. We prove properties to the components of the solutions as t approaches infinity.

Mathematics subject classification (2010): 34K11, 34K25, 34K40.
Keywords and phrases: neutral differential equation, oscillatory solution, asymptotic properties of solutions.

REFERENCES

[1] Z. CHEN, T. SUN, Q. WANG, H. XI, *Nonoscillatory Solutions for System of Neutral Dynamic Equations on Time Scales*, The scientific World Journal, Volume 2014, (2014), Article ID 768215, 10 pages http://dx.doi.org/10.1155/2014/768215

[2] J. DIBLÍK, M. KÚDELČÍKOVÁ, *Existence and asymptotic behavior of positive solutions of functional differential equations of delayed type*, Abstr. Appl. Anal., Vol. 2011, Article ID 754701, 16 pages, (2011).

[3] J. DIBLÍK, M. RUŽIČKOVÁ, Z. ŠUTÁ, *Asymptotical convergence of the solutions of a linear differential equation with delays*, Adv. Difference Equ., Vol. 2010, Article ID 749852, 12 pages, (2010).

[4] J.G. DIX, *Oscillation of solutions to a neutral differential equation involving an n-order operator with variable coefficients and a forcing term*, Differ. Equ. Dynamic Systems, DOI 10.1007/s12591-013-0160-z.

[5] B. DOROCIAKOVÁ, M. KUBJATKOVÁ, R. OLACH, *Existence of positive solutions of neutral differential equations*, Abstr. Appl. Anal., Vol. 2012, ID 307968, 14 pages, (2012).

[6] J. DŽURINA, B. BACULÍKOVÁ, *Oscillation and Asymptotic Behavior of Higher-Order Nonlinear Differential Equations*, Int. J. Math. Mathematical Sciences, (2012), 2012.

[7] L.H. ERBE, Q. KONG, B.G. ZHANG, *Oscillatory theory for functional differential equations*, Marcel Dekker, 1995.

[8] Y. KITAMURA, T. KUSANO, *Asymptotic properties of solutions of two-dimensional differential systems with deviating argument*, Hiroshima Math. J., Vol. 8 (1978), 305–326.

[9] P. MARUŠIAK, *On unbounded nonoscillatory solutions of systems of neutral differential equations*, Czech. Math. J., 42, No.1 (1992), 117–128.

[10] I. MOJSEJ, J. OHRISKA, *Comparison theorems for noncanonical third order nonlinear differential equations*, Cent. Eur. J. Math., Volume 5, No. 1 (2007), 154–163.

Paper DEA-07-01

[11] H. ŠAMAJOVÁ, E. ŠPÁNIKOVÁ, J.G. DIX, *Decay of non-oscillatory solutions for a system of neutral differential equations*, Electronic Journal of Differential Equations, 2013, No. 271 (2013), 1–11. http://ejde.math.txstate.edu

[12] Z. ŠMARDA, J. REBENDA, *Asymptotic Behaviour of a Two-Dimensional Differential System with a Finite Number of Nonconstant Delays under the Conditions of Instability*, Abstr. Appl. Anal., Vol. 2012, Special Issue (2012)

[13] E. ŠPÁNIKOVÁ, H. ŠAMAJOVÁ, *Asymptotic properties of solutions to n-dimensional neutral differential systems*, Nonlinear Analysis, 71 (2009), 2877–2885.

[14] S. STANĚK, *Oscillation behaviour of solutions of neutral delay differential equations*, Čas. Pěstování Mat., Vol. 115, No.1 (1990), 92–99.

[15] J. REBENDA, Z. ŠMARDA, *Stability and asymptotic properties of a system of functional differential equations with nonconstant delays*, Appl. Math. Comput., 219 (2013), 6622–6632.