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EVOLUTION EQUATIONS WITH CAUSAL OPERATORS

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Abstract. In this paper we present an existence result for causal functional evolution equations. The result is obtained under a condition with respect to the Hausdorff measure of noncompactness. An application with partial differential equations is given to illustrate our main result.

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REFERENCES

[1] R.P. AGARWAL, Y. ZHOU, J.R. WANG AND X. LUO, *Fractional functional differential equations with causal operators in Banach spaces*, Mathematical and Computer Modelling, **54** (2011), 1440–1452.
[2] D.M. BEDIVAN AND D. O'REGAN, *The set of solutions for abstract Volterra equations in* $L^p([0,a],\mathbb{R}^m)$, Applied Mathematics Letters, **12** (1999), 7–11.
[3] S. ARSHAD, V. LUPULESCU, *On a class of controlled functional differential inclusions*, Le Matematiche, **68** (2013), 53–64.
[4] R. AHANGAR, *Nonanticipating dynamical model and optimal control*, Applied Mathematics Letters, **2** (1989), 15–18.
[5] O. CÂRJĂ, M. NECULA, I.I. VRABIE, *Viability, Invariance and Applications*, North-Holland Mathematics Studies, 207, Elsevier Science B.V., Amsterdam, 2007.
[6] C. CORDUNEANU, *Functional Equations with Causal Operators*, Taylor and Francis, London and New York, 2002.
[7] C. CORDUNEANU, *A modified LQ-optimal control problem for causal functional differential equations*, Nonlinear Dynamics and Systems Theory, **4** (2004), 139–144.
[8] C. CORDUNEANU AND M. MAHDAVI, *Neutral functional equations with causal operators on a semi-axis*, Nonlinear Dynamics and Systems Theory, **8** (2008), 339–348.
[9] Z. DRICI, F.A. MCRAE AND J.V. DEVI, *Monotone iterative technique for periodic boundary value problems with causal operators*, Nonlinear Analysis: Theory, Methods & Applications, **64** (2006), 1271–1277.
[10] Z. DRICI, F.A. MCRAE AND J. V. DEVI, *Differential equations with causal operators in a Banach space*, Nonlinear Analysis: Theory, Methods & Applications, **62** (2005), 301–313.
[11] M. I. GIL, *Positive solutions of equations with nonlinear causal mappings*, Positivity, **11** (2007), 523–535.
[12] G. GRIPENBERG, S.O. LONDEN AND O. STAFFANS, *Volterra Integral and Functional Equations*, Cambridge University Press, 1990.
[13] E. HERNANDEZ, D. O'REGAN AND M. A. BEN, *On a new class of abstract integral equations and applications*, Applied Mathematics and Computation, **219**, 4 (2012), 2271–2277.
[14] A. ILCHMANN, E.P. RYAN AND C.J. SANGWIN, *Systems of controlled functional differential equations and adaptive tracking*, Siam Journal on Control and Optimization, **40**, 6 (2002), 1746–1764.
[15] T. JANKOWSKI, *Nonlinear boundary value problems for second order differential equations with causal operators*, Journal of Mathematical Analysis and Applications, **332** (2007), 1380–1392.
[16] M. KAMENSKII, V. OBUKHOVSKII AND P. ZECCA, *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces*, de Gruyter Series in Nonlinear Analysis and Applications, vol. 7, Walter de Gruyter, Berlin, New York, 2001.

Paper DEA-07-02

[17] G. KARAKOSTAS, *Uniform asymptotic stability of causal operator equations*, Journal of Integral Equations, **5** (1983), 59–71.
[18] M. KISIELEWICZ, *Multivalued differential equations in separable Banach spaces*, Journal of Optimization Theory and Applications, **37**, 2 (1982), 231–249.
[19] V. LAKSHMIKANTHAM AND S. LEELA, *Nonlinear Differential Equations in Abstract Spaces*, Pergamon Press, New York, 1969.
[20] V. LAKSHMIKANTHAM, S. LEELA, Z. DRICI AND F.A. MCRAE, *Theory of Causal Differential Equations*, Atlantis Studies in Mathematics for Engineering and Science, vol. 5, World Scientific, 2010.
[21] J. H. LIU, G. M. N'GUEREKATA AND N. V. MINH, *Topics on Stability and Periodicity in Abstract Differential Equations*, World Scientific Publishing, Singapore, 2008.
[22] V. LUPULESCU, *Causal functional differential equations in Banach spaces*, Nonlinear Analysis, **69** (2008), 4787–4795.
[23] V. LUPULESCU, *Periodic boundary value problem for impulsive differential equations with causal operators*, Nonlinear Studies, **17** (2010), 151–162.
[24] V. LUPULESCU, *On a class of functional differential equations in Banach spaces*, Electronic Journal of Qualitative Theory of Differential Equations, **64** (2010), 1–17.
[25] V. LUPULESCU, *Functional differential equations with causal operators*, International Journal of Nonlinear Science, **11** (2011), 499–505.
[26] M. MAHDAVI, *Linear functional differential equations with abstract Volterra operators*, Differential and Integral Equations, **8** (1995), 1517–1523.
[27] A. MCNABB, *An initial value theory for linear causal boundary value problems*, Journal of Differential Equations, **15** (1974), 322–349.
[28] V. OBUKHOVSKII AND P. ZECCA, *On certain classes of functional inclusions with causal operators in Banach spaces*, Nonlinear Analysis: Theory, Methods & Applications, **74** (2011), 2765–2777.
[29] A. PAZY, *Semigroups of Linear Operators and Applications for Partial Differential Equations*, Springer-Verlag, New York, 1983.
[30] D. O'REGAN, *A note on the topological structure of the solutions set of abstract Volterra equations*, Proceedings of the Royal Irish Academy - Section A: Mathematical and Physical Sciences, **99** (1999), 67–74.
[31] D. O'REGAN AND R. PRECUP, *Existence criteria for integral equations in Banach spaces*, Journal of Inequalities and Applications, **6** (2001), 77–97.
[32] E.P. RYAN AND C.J. SANGWIN, *Controlled functional differential equations and adaptive tracking*, Systems Control Letters, **47** (2002), 365–374.
[33] S. SZUFLA, *On the existence of L^p-solutions of Volterra integral equations in Banach spaces*, Funkcialaj Ekvacioj, **27** (1984), 157–172.
[34] A. N. TIKHONOV, *Functional Volterra-type equations and their applications to certain problems of mathematical physics*, Bull.Mosk. Gos. Univ., Sekt. A., **1**, 8 (1938), 1–25.
[35] L. TONELLI *Sulle equazioni funzionali di Volterra*, Bulletin of Calcutta Mathematical Society, **20** (1930), 31–48.
[36] C. C. TRAVIS AND G. F. WEBB, *Existence and stability for partial functional differential equations*, Transactions of the American Mathematical Society, **200** (1974), 395–418.
[37] I. I. VRABIE, *C_0-semigroups and Applications*, North-Holland Publishing Co., Amsterdam, 2003.
[38] E.S. ZHUKOVSKII AND M.J. ALVES, *Abstract Volterra Operators*, Russian Mathematics, **52** (2008), 1–14.