

A GENERALIZATION OF DARBO'S FIXED POINT THEOREM AND LOCAL ATTRACTIVITY OF GENERALIZED NONLINEAR FUNCTIONAL INTEGRAL EQUATIONS

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Abstract. We prove a generalization of a measure theoretic fixed point theorem of Darbo in Banach spaces which includes some well-known fixed point theorems of Dhage and Sadovskii as special cases. A generalized nonlinear functional integral equation is studied via Dhage fixed point theorem for attractivity of the solutions on unbounded intervals of real line. Finally the validity of our hypotheses imposed on the functional integral equation is also discussed with a numerical example.

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