Differential
Equations
& Applications
Volume 7, Number 1 (2015), 79–91
doi:10.7153/dea-07-06

EXISTENCE AND ASYMPTOTIC BEHAVIOR OF STRONGLY MONOTONE SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS

TOMOYUKI TANIGAWA

Abstract. Two types of nonlinear differential systems

$$\text{(A)}\quad x' + p(t)y^{\alpha} = 0,\quad y' + q(t)x^{\beta} = 0; \qquad \text{(B)}\quad x' - p(t)y^{\alpha} = 0,\quad y' - q(t)x^{\beta} = 0$$

are considered under the assumption that α and β are positive constants such that $\alpha\beta < 1$ and $p(t)$ and $q(t)$ are continuous regularly varying functions on a neighborhood of infinity. An attempt is made to obtain precise information on the existence and asymptotic behavior of strongly monotone regularly varying solutions $(x(t), y(t))$ of (A) and (B) whose x-components or y-components are slowly varying. It is shown that the results thus obtained are applied to the generalized Thomas-Fermi equations of the form $(p(t)|x'|^{\alpha-1}x')' = q(t)|x|^{\beta-1}x$ to provide new useful knowledge of their strongly monotone solutions. The present paper is designed to supplement the pioneering results on the asymptotic analysis of (A) and (B) by means of regular variation developed in the paper [4].

Mathematics subject classification (2010): 34C11, 26A12.
Keywords and phrases: systems of differential equations, asymptotic behavior, regularly varying functions.

REFERENCES

[1] N. H. BINGHAM, C. M. GOLDIE AND J. L. TEUGELS, *Regular Variation*, Encyclopedia of Mathematics and its Applications, **27**, Cambridge University Press, 1987.
[2] E. FERMI, *Un metodo statistico par la determinazione di alcune proprietà dell'atome*. Rendiconti Accademia Nazionale del Lincei. Cl. sci. fis., mat. e nat., **6**, (6) (1927), 602 – 607.
[3] E. HILLE, *Some aspects of the Thomas – Fermi equation*, J. Analyse Math., **23** (1970), 147 – 170.
[4] J. JAROŠ AND T. KUSANO, *Existence and precise asymptotic behavior of strongly monotone solutions of systems of nonlinear differential equations*, Differ. Equ. Appl., **5** (2013), 185 – 204.
[5] J. JAROŠ AND T. KUSANO, *Slowly varying solutions of a class of first order systems of nonlinear differential equations*, Acta. Math. Univ. Comenianae, **82** (2013), 265 – 284.
[6] T. KUSANO, V. MARIĆ AND T. TANIGAWA, *An asymptotic analysis of positive solutions of generalized Thomas-Fermi differential equations – The sublinear case*, Nonlinear Anal., **75** (2012), 2474 – 2485.
[7] L. H. THOMAS, *The calculation of atomic fields*. Proceedings Cambridge Philosophical Society, **23** (1927), 542 – 548.