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THE RICCATI EQUATION METHOD WITH VARIABLE EXPANSION COEFFICIENTS. III. SOLVING THE NEWELL–WHITEHEAD EQUATION

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Abstract. The Riccati equation method with variable expansion coefficients, introduced in previous papers, is used to find traveling wave solutions to the Newell-Whitehead (NW) equation $u_t = u_{xx} + au - bu^3$. The ξ-dependent coefficients A and B of the Riccati equation $Y' = A + BY^2$ are either proportional each other or their product is equals to an exponential function. They are determined as solutions of ODEs they satisfy and their solutions are expressed either in terms of Bessel's functions or in terms of functions already found in Paper I. The same situation occurs for the expansion coefficients as well. The function Y which is a solution of Riccati's equation, is expressed in terms of Bessel functions or it is a constant quantity.

Mathematics subject classification (2010): 35C05, 35C07, 35K57.
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