

BOUNDARY BEHAVIOR OF LARGE SOLUTIONS TO QUASILINEAR ELLIPTIC PROBLEMS WITH A NONLINEAR GRADIENT TERM

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Abstract. In this paper, we study the boundary behavior of solutions to boundary blow-up elliptic problems

$$\begin{cases} \operatorname{div}(|\nabla u|^{m-2} \nabla u \pm |\nabla u(x)|^{q(m-1)} = b(x)f(u), & x \in \Omega, \\ u > 0, & x \in \Omega, \\ u|_{\partial\Omega} = +\infty, \end{cases}$$

where Ω is a bounded domain with smooth boundary in \mathbb{R}^N , $m > 1$, $q > 0$, $b \in C^\alpha(\overline{\Omega})$, which is positive in Ω and may be vanishing on the boundary and rapidly varying near the boundary, and f is rapidly varying or normalized regularly varying at infinity.

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