

GLOBAL EXISTENCE OF RADIAL SOLUTIONS OF A HYPERBOLIC MEMS EQUATION WITH NONLOCAL TERM

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Abstract. We consider a nonlocal hyperbolic MEMS equation in the higher dimensional annular domain. In this paper, we concentrate on the radial solutions. First we establish a time-local solution by a contraction mapping theorem. This procedure is standard. Next we show that there exists a global solution for small parameter and initial value. The important facts for the proof are the Sobolev embedding theorem and the energy conservation. Finally, we deal with the corresponding stationary problem. By the maximum principle, we can evade integrating the stationary solution over the domain near the boundary. Then we establish the upper bound of the parameter for the existence of the stationary solution.

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