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BLOWUP ESTIMATES FOR A FAMILY OF SEMILINEAR SPDES WITH TIME–DEPENDENT COEFFICIENTS

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Abstract. We investigate the blowup and stability of semilinear stochastic partial differential equations with time-dependent coefficients using stopping times of exponential functionals of Brownian martingales and a non-homogeneous heat semigroup. In particular we derive lower bounds for the probability of blowup in finite time, and we provide sufficient conditions for the existence of global positive solutions.

Mathematics subject classification (2010): 60H15, 35R60, 35K58, 35B40, 35B44.
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