Differential
Equations
& Applications
Volume 7, Number 2 (2015), 221–244
doi:10.7153/dea-07-13

NONLINEAR SCHRÖDINGER EQUATION WITH LANDAU DAMPING ON A HALF–LINE

LILIANA ESQUIVEL

Abstract. We consider the initial-boundary value problem for the modified Schrödinger equation, posed on positive half-line $x > 0$:

$$\begin{cases} u_t + \mathbb{K}u + i|u|^2 u = 0,\ t \geqslant 0,\ x \geqslant 0; \\ u(x,0) = u_0(x),\ \ x > 0 \\ u(0,t) = h(t),\ t > 0. \end{cases}$$

where the operator $\mathbb{K}$ is defined as

$$\mathbb{K}(u) = \alpha u_{xx} + \lambda |\partial_x|^\gamma u$$

with $\alpha \in \mathbb{C}$, $\lambda > 0$ and $|\partial_x|^\gamma$ is the module-fractional derivative operator defined by

$$|\partial_x|^\gamma u = R^\gamma \partial_x u.$$

Here R^γ is the modified Riesz Potential

$$R^\gamma u = \frac{1}{2\sqrt{\pi}\sin\left(\frac{\pi}{4}\right)} \int_0^\infty \frac{sign\,(x-y)}{\sqrt{|x-y|}} u(y)dy.$$

We study the local and global existence in time of solutions to the initial-boundary value problem.

Mathematics subject classification (2010): 35Q55, 35B40.
Keywords and phrases: fractional Schrödinger equation, boundary value, Landau damping.

REFERENCES

[1] M.P. ÁRCIGA, *Asymptotics for Nonlinear Evolution Equation with Module-Fractional Derivative on a Half-Line*, Boundary Value Problem, vol. 2011, Article ID 946143, 29 pages, 2011.
[2] F. BENITEZ, E. KAIKINA, *A Neumann problem for the KdV equation with Landau damping on a half-line*, Nonlinear Analysis, 74 (2011), 4682–4697.
[3] T. CAZENAVE, Semilinear Schrödinger Equations. American Mathematical Soc., 2003.
[4] A. DEGASPERIS, S.V. MANAKOV, P.M. SANTINI, *On the initial-boundary value problems for soliton equations,* Journal of Experimental and Theoretical Physics Letters 2001, Volume 74, Issue 10, pp 481–485.
[5] A.S. FOKAS, *Integrable nonlinear evolution equations on the half-line*, Commun. Math. Phys., 230 (2002), 1–39.
[6] A.S. FOKAS, A.R. Its, L-Y Sung. *The nonlinear Schrödinger equation on the half-line*, Nonlinearity 18, (4) (2005), 1771–1822.
[7] F.D. GAKHOV, *Boundary Value Problems*, Dover Publications, INC. New York, 1966.
[8] X. GUO, M. XU, *Some physical applications of fractional Schrödinger equation*, Journal of Mathematical Physics, 47 (2006), 082104.
[9] B. GUO, Z. HUO, *Global Well-Posedness for the Fractional Nonlinear Schrödinger Equation,* Communications in Partial Differential Equations, (2010), 247–255.

Paper DEA-07-13

[10] A.D. IONESCU, F. PUSATERI, *Nonlinear fractional Schrödinger equations in one dimension*, Journal of Functional Analysis, 266 (2014).
[11] N. HAYASHI, E. KAIKINA, *Nonlinear theory of pseudodifferential equations on a half-line*. North-Holland Mathematics Studies, 194. Elsevier Science B. V., Amsterdam, 2004, 319 pp.
[12] N. HAYASHI, P.I. NAUMKIN, *Asymptotics of small solutions to nonlinear Schrödinger equations with cubic nonlinearities*. Int. J. Pure Appl. Math 3, no. 3 (2002), 255–273.
[13] E. KAIKINA, *Fractional derivative of Abel type on a Half Line*. Transactions of the American Mathematical Society. Vol. 364, No. 10, October 2012, Pages 5149–5172.
[14] E.I. KAIKINA, *Asymptotics for inhomogeneous Dirichlet initial-boundary value problem for the nonlinear Schrödinger equation,* J. Math. Phys., 54 (2013), no. 11 111504, 15 pp.
[15] N. LASKIN, *Fractional Schrödinger equation*
[16] E.K. LENZI, H.V. RIBEIRO, M.A.F. DOS SANTOS, R. ROSSATO, R. S. MENDES, *Time dependent solutions for a fractional Schrödinger equation with delta potentials*, Journal of Mathematical Physics, 54, 2013.
[17] S. WANG, M. XU, *Generalized fractional Schrödinger equation with space time fractional derivatives*, Journal of Mathematical Physics, 48, no. 4, 043502, (2007), 10 pp.
[18] D. WU, *Existence and stability of standing waves for nonlinear fractional Schrödinger equations with Hartree type nonlinearity* J. Math. Anal. Appl., 411 (2014), 530–542.