

## EXISTENCE AND UNIQUENESS OF POSITIVE SOLUTIONS TO A COUPLED SYSTEM OF NONLINEAR FRACTIONAL ORDER DIFFERENTIAL EQUATIONS WITH ANTI PERIODIC BOUNDARY CONDITIONS

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*Abstract.* In this article, we study sufficient conditions for existence and uniqueness of positive solutions to the following coupled system of fractional order differential equations with anti-periodic boundary conditions

$$\begin{cases} {}^c D^\alpha u(t) + f(t, v(t), {}^c D^{\alpha-1} v(t)) = 0, {}^c D^\beta v(t) + g(t, u(t), {}^c D^{\beta-1} u(t)) = 0, & 0 < t < 1, \\ u(0) = -u(1), v(0) = -v(1), D^p u(0) = -D^p u(1), D^q v(0) = -D^q v(1), \end{cases}$$

where  $1 < \alpha, \beta \leq 2, \alpha - p \geq 1, \beta - q \geq 1$  and  $0 < p, q < 1, f, g : [0, 1] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  are continuous functions and  $D$  stands for Caputo derivative. We use Banach and Schauder fixed point theorems to develop sufficient conditions for existence and uniqueness of positive solutions. We also study sufficient conditions for existence of multiple positive solutions and conditions for non existence of solutions. We provide several examples to show the applicability of our results. We also link our analysis for the problem to equivalent integral equations.

*Mathematics subject classification* (2010): 47J05, 92D25, 34A08, 34A34, 34K15.

*Keywords and phrases:* Coupled system, Fractional differential equations, Anti-periodic boundary conditions, Fixed point theorems.

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