Differential
Equations
& Applications
Volume 7, Number 2 (2015), 263–276
doi:10.7153/dea-07-15

GLOBAL EXISTENCE AND BLOW–UP FOR NONAUTONOMOUS SYSTEMS WITH NON–LOCAL SYMMETRIC GENERATORS AND DIRICHLET CONDITIONS

AROLDO PÉREZ

Abstract. We study a semilinear system of the form

$$\frac{\partial u_i(t,x)}{\partial t} = k_i(t)\mathscr{A}_i u_i(t,x) + u_{i'}^{\beta_i}(t,x), \quad t>0, \ \ x\in D,$$
$$u_i(0,x) = f_i(x), \quad x\in D, \quad u_i|_{D^c}\equiv 0,$$

where $D\subset\mathbb{R}^d$ is a bounded open domain, $k_i:[0,\infty)\to[0,\infty)$ is continuous, $\mathscr{A}_i$ is the infinitesimal generator of a symmetric jump-type process $Z_i\equiv\{Z_i(t)\}_{t\geqslant 0}$, $\beta_i>1$, $i\in\{1,2\}$ and $i'=3-i$. Under some assumptions on the infinitesimal generator $\mathscr{A}_i^D$ of the subprocess Z_i killed upon leaving D, $i=1,2$, we give sufficient conditions for global existence or finite-time blow-up of the positive mild solutions of our system. This paper can be considered as a continuation of the article [16].

Mathematics subject classification (2010): 60J75, 60G52, 35B35, 35K57.
Keywords and phrases: semilinear nonautonomous equations, Dirichlet problem, ultracontractive semigroup, Markov evolution systems, finite-time blow up.

REFERENCES

[1] Z-Q. CHEN, P. KIM AND R. SONG, *Sharp heat kernel estimates for relativistic stable processes in open sets*, Ann. Probab., **40**, 1 (2012), 213–244.

[2] Z-Q. CHEN, P. KIM AND R. SONG, *Dirichlet Heat Kernel Estimates for* $\Delta^{\frac{\alpha}{2}}+\Delta^{\frac{\beta}{2}}$, Illinois J. Math., **54**, 4 (2010), 1357–1392.

[3] Z-Q. CHEN, P. KIM AND R. SONG, *Heat kernel estimates for the Dirichlet fractional Laplacian*, J. Eur. Math. Soc., **12** (2010), 1307–1329.

[4] Z-Q. CHEN AND T. KUMAGAI, *Heat kernel estimates for jump processes of mixed types on metric measure spaces*, Probab. Theory Relat. Fields, **140** (2008), 277–317.

[5] K.DENG AND H. A. LEVINE, *The role of critical exponents in blow-up theorems: the sequel*, J. Math. Anal. Appl., **243**, 1 (2000), 85–126.

[6] M. ESCOBEDO AND M. A. HERRERO, *Boundedness and blow up for a semilinear reaction-fiffusion system*, J. Differential Equations, **89**, 1 (1991), 176–202.

[7] H. FUJITA, *On some nonexistence and nonuniqueness theorems for nonlinear parabolic equations*, *Nonlinear Functional Analysis (Proc. Sympos. Pure Math., Vol. XVIII, Part 1, Chicago, Ill., 1968)*, Amer. Math. Soc., Providence, R. I. (1970), 105–113.

[8] V. A. GALAKTIONOV, S. P. KURDYUMOV AND A. A. SAMARSKII, *A parabolic system of quasilinear equations. I.*, Differential Equations, **19**, 12 (1983), 1558–1574.

[9] V. A. GALAKTIONOV, S. P. KURDYUMOV AND A. A. SAMARSKII, *A parabolic system of quasilinear equations. II.*, Differential Equations, **21**, 9 (1985), 1049–1062.

[10] T. GRZYWNY, *Intrinsic ultracontractivity for Lévy processes*, Prob. Math. Stat., **28**, 1 (2008), 91–106.

[11] M. GUEDDA AND M. KIRANE, *Critically for some evolution equations*, Differential Equations, **37**, 4 (2001), 540–550.

Paper DEA-07-15

[12] S. KERBAL, *Non-existence of global solutions to systems of non-autonomous nonlinear parabolic equations*, Commun. Appl. Anal., **14**, 2 (2010), 203–212.

[13] M. KIRANE, Y. LASKRI AND N. TATAR, *Critical exponentes of Fujita type for certain evolution equations and systems with spatio-temporal fractional derivatives*, J. Math. Anal. Appl., **312** (2005), 488–501.

[14] M. KIRANE AND M. QAFSAOUI, *Global nonexistence for the Cauchy problem of some nonlinear reaction-diffusion systems*, J. Math. Anal. Appl., **268** (2002), 217–243.

[15] E. T. KOLKOVSKA, J. A. LÓPEZ-MIMBELA AND A. PÉREZ, *Blow-up and life span bounds for a reaction-diffusion equation with a time-dependent generator*, Elec. J. Diff. Equations, **2008**, 10 (2008), 1–18.

[16] J. A. LÓPEZ-MIMBELA AND A. PÉREZ, *Global and nonglobal solutions of a system of nonautonomous semilinear equations with ultracontractive Lévy generators*, J. Math. Anal. Appl., **423**, 1 (2015), 720-733.

[17] J. A. LÓPEZ-MIMBELA AND A. PÉREZ, *Finite time blow up and stability of a semilinear equation with a time dependent Lévy generator*, Stoch. Models, **22**, 4 (2006), 735-752.

[18] J. A. LÓPEZ-MIMBELA AND A. TORRES, *Intrinsic ultracontractivity and blowup of a semilinear Dirichlet boundary value problem*, Aportaciones Matemáticas, Modelos Estocásticos, **14**, Sociedad Matemática Mexicana, (1998), 283-290.

[19] A. PÉREZ, *A blow up condition for a nonautonomous semilinear system*, Electron. J. Diff. Eqns., **2006**, 94 (2006), 1–8.

[20] A. PÉREZ AND J. VILLA, *A note on blow-up of nonlinear integral equation*, Bull. Belg. Math. Soc.-Simon Stevin, **17** (2010), 891–897.

[21] M. RYZNAR, *Estimates of Green function for relativistic α-stable process*, Potential Anal., **17** (2002), 1–23.

[22] Y. UDA, *The critical exponent for a weakly coupled system of the generalized Fujita type reaction-diffusion equations*, Z. ang. Math. Phys., **46** (1995), 366–383.

[23] J. VILLA-MORALES, *Blow up of mild solutions of a system of partial differential equations with distinct fractional diffusions*, Electron. J. Diff. Eqns., **2014**, 41 (2014), 1–9.