EXISTENCE OF SOLUTIONS AND SEMI–DISCRETIZATION
FOR PDE WITH INFINITE DELAY

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Abstract. In this paper, we obtain an existence theorem for a Semi-Linear PDE with infinite delay employing a phase space in which discretizations can naturally be performed. Further, for linear PDEs with infinite delay we show that the solutions of the ODE with infinite delay obtained by the semi-discretization converge to the original solution. Our results cover various types of PDEs under the assumption that semi-discretization of the PDEs without the delay terms can be performed. The method of our proof is applicable for the case of finite delays too.

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