Differential
Equations
& Applications
Volume 7, Number 3 (2015), 363–384
doi:10.7153/dea-07-21

OSCILLATION CRITERIA FOR ODD HIGHER ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS WITH POSITIVE AND NEGATIVE COEFFICIENTS

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Abstract. In this paper, the authors study oscillatory and asymptotic behavior of solutions of a class of nonlinear higher order neutral differential equations with positive and negative coefficients of the form

$$(a(t)(b(t)(y(t)+p(t)y(\sigma(t)))')')^{(n-2)}+q(t)G(y(\alpha(t)))-h(t)H(y(\beta(t)))=0 \qquad \text{(E)}$$

for $n\geqslant 3$, n is an odd integer, $0\leqslant p(t)\leqslant p_1<1$ and $-1<p_2\leqslant p(t)\leqslant 0$. The results in this paper generalize the results of Panigrahi and Basu [9] and various results in the literature. We establish new conditions which guarantees that every solutions of (E) either oscillatory or converges to zero. Examples are considered to illustrate the main results.

Mathematics subject classification (2010): 34C10, 34C15, 34K11.
Keywords and phrases: functional differential equations, neutral, nonlinear, oscillation, positive and negative coefficients.

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