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SYMMETRIC POSITIVE SOLUTIONS FOR SECOND–ORDER SINGULAR DIFFERENTIAL SYSTEMS WITH MULTI–POINT COUPLED INTEGRAL BOUNDARY CONDITIONS

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Abstract. In this paper, we study the existence and multiplicity of symmetric positive solutions for a class of second-order singular nonlinear differential systems with multi-point coupled integral boundary conditions. By constructing a special cone and applying the fixed point theorem of cone expansion and compression of norm type, the existence results of single and multiple symmetric positive solutions are established. As applications, two examples are given to demonstrate the applicability of our results.

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