

## POSITIVE SOLUTIONS FOR A SINGULAR THIRD ORDER BOUNDARY VALUE PROBLEM

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*Abstract.* The existence of positive solutions is shown for the third order boundary value problem,  $u''' = f(x, u)$ ,  $0 < x < 1$ ,  $u(0) = u(1) = u''(1) = 0$ , where  $f(x, y)$  is singular at  $x = 0$ ,  $x = 1$ ,  $y = 0$ , and may be singular at  $y = \infty$ . The method involves application of a fixed point theorem for operators that are decreasing with respect to a cone.

*Mathematics subject classification* (2010): 34B16, 34B18.

*Keywords and phrases:* fixed point theorem; boundary value problem, singular.

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