

## SCALAR MULTI-POINT BOUNDARY VALUE PROBLEMS AT RESONANCE

DANIEL MARONCELLI

**Abstract.** In this paper we discuss the solvability of multi-point boundary value problems of the form

$$y^{(n)}(t) + a_{n-1}(t)y^{(n-1)}(t) + \cdots + a_0(t)y(t) = g(t, y(t))$$

subject to

$$\sum_{j=1}^n b_{ij}(0)y^{(j-1)}(t_0) + \sum_{j=1}^n b_{ij}(1)y^{(j-1)}(t_1) + \cdots + \sum_{j=1}^n b_{ij}(k)y^{(j-1)}(t_k) = 0$$

for  $i = 1, \dots, n$ .

We improve upon existing results in the literature regarding multi-point boundary value problems. Our approach uses an alternative method along with Schaefer's fixed point theorem.

*Mathematics subject classification* (2010): 34B10.

*Keywords and phrases:* multi-point boundary value problems, resonance, Lyapunov-Schmidt procedure, Schaefer's Fixed Point Theorem.

### REFERENCES

- [1] Z. DUA, X. LIN, AND W. GE, *Some higher-order multi-point boundary value problems at resonance*, J. Comput. Appl. Math., **177** (2005), 55–65.
- [2] M. FENG AND W. GE, *Existence results for a class of  $n$ th order  $m$ -point boundary value problems in banach spaces*, Appl. Math. Lett., **22** (2009), 1303–1308.
- [3] M. HILAL, *Multi-point boundary value problems*, Lambert Academic Publishing, 2012.
- [4] J. HENDERSON AND R. LUCA, *Positive solutions for system of nonlinear second-order multipoint boundary value problems*, Math. Meth. Appl. Sci., **37** (2014), 2502–2516.
- [5] W. JIANG, B. WANG, AND Z. WANG, *Solvability of a second-order multi-point boundary-value problems at resonance on a half-line with  $\dim \ker l=2$* . Electron. J. Differential Equations, **2001**, (2011), 1–11.
- [6] CHAN-GYUN KIM, *Solvability of multi-point boundary value problems on the half-line*. J. Nonlinear Sci. Appl., **5** (2012), 27–33.
- [7] S. LIANG AND J. ZHANG, *The method for lower and upper solution to  $2n$ th-order multi-point boundary value problems*, Nonlinear Anal., **71**, (2009), 4581–4587.
- [8] XINAN HAO LISHAN LIU AND YONGHONG WU, *Multi-point boundary value problems for higher order differential equations*, Appl. Math. E-Notes, **4** (2004), 106–113.
- [9] X LIU AND W LI, *Positive solutions for the nonlinear fourth-order beam equations with three parameters*, J. Math. Anal. Appl., **303** (2005), 150–163.
- [10] R. MA, *Multiple positive solutions for nonlinear  $m$ -point boundary value problems*, Appl. Math. Comput., **148** (2004), 249–262.
- [11] D. MARONCELLI AND J. RODRÍGUEZ, *On the solvability of multipoint boundary value problems for discrete systems at resonance*, J. Difference Equ. Appl., **20**, Issue 1 (2013), 24–35.
- [12] D. MARONCELLI AND J. RODRÍGUEZ, *A least squares solution to linear boundary value problems with impulses*, Differ. Equ. Appl., **5**, Issue 4 (2013), 519–525.

- [13] D. MARONCELLI AND J. RODRÍGUEZ, *Weakly nonlinear boundary value problems with impulses*, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal., **20** (2013), 641–656.
- [14] D. MARONCELLI AND J. RODRÍGUEZ, *Existence theory for nonlinear Sturm-Liouville problems with unbounded nonlinearities*. Differ. Equ. Appl., **6** (2014), 455–466.
- [15] D. MARONCELLI AND J. RODRÍGUEZ, *On the solvability of nonlinear impulsive boundary value problems*, Topol. Methods Nonlinear Anal., **44** (2015), 381–398.
- [16] M. MOSHINSKY, *Sobre los problemas de condiciones a la frontera en una dimensión de características discontinuas*, Bol. Soc. Mat. Mexicana, **7** (1950), 1–25.
- [17] JESÚS RODRÍGUEZ AND PADRAIC TAYLOR, *Multipoint boundary value problems for nonlinear ordinary differential equations*, Nonlinear Anal., **68** (2008), 3465–3474.
- [18] S. TIMOSHENKO, *Theory of elastic stability*, McGraw-Hill, New York, 1961.
- [19] Y. M. WANG, *The iterative solutions to  $2n$ th-order nonlinear multi-point boundary value problems*, Appl. Math. Comput., **217** (2010), 2251–2259.
- [20] J.S.W. WONG AND L. KONG, *Positive solutions for higher order multi-point boundary value problems with nonhomogeneous boundary conditions*, J. Math. Anal. App., **367** (2010), 367–588.
- [21] WEI-HUA JIANG XIU-JUN LIU AND YAN PING GUO, *Multi-point boundary value problems for higher order differential equations*, Appl. Math. E-Notes, **4** (2004), 106–113.