DYNAMICAL BEHAVIOR OF A STAGE STRUCTURED ECO–EPIDEMIOLOGICAL MODEL

SHASHI KANT AND VIVEK KUMAR

Abstract. In this paper, a stage structured eco-epidemiological model with linear functional response is proposed and studied. The stages for both prey and predator have been considered. Infection occurs in the prey population only. The proposed mathematical model consists of five nonlinear ordinary differential equations to describe the interaction among juvenile prey, adult prey, infected prey, juvenile predator and adult predator populations. The model is analyzed by using linear stability analysis to obtain the conditions for which our model exhibits stability around the possible equilibrium points.

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