

## INFINITELY MANY SOLUTIONS FOR A FOURTH-ORDER NONLINEAR ELLIPTIC SYSTEM

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*Abstract.* In this paper we study the existence of solutions for the nonlinear elliptic system

$$\begin{cases} \Delta^2 u - \Delta u + V_1(x)u = f_u(x, u, v), \\ \Delta^2 v - \Delta v + V_2(x)v = f_v(x, u, v), \\ u, v \in H^2(\mathbb{R}^N) \quad x \in \mathbb{R}^N, \end{cases}$$

where  $V_1(x)$  and  $V_2(x)$  are positive continue functions. Under some assumptions on  $f_u(x, u, v)$  and  $f_v(x, u, v)$ , we prove the existence of many nontrivial high and small energy solutions by variant Fountain theorems. This generalizes the results by Y. Ye and C. Tang (J. Math. Anal. Appl. 394, 841-854, 2012) to fourth-order nonlinear elliptic system.

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