

EXISTENCE OF POSITIVE SOLUTION AND NEW OSCILLATION CRITERIA FOR NONLINEAR FIRST-ORDER NEUTRAL DELAY DIFFERENTIAL EQUATIONS

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Abstract. In this work, oscillatory behaviour of the solutions of a class of nonlinear first-order neutral delay differential equations of the form

$$(E_1) \quad (x(t) + p(t)x(t - \tau))' + q(t)H(x(t - \sigma)) = f(t)$$

and

$$(E_2) \quad (x(t) + p(t)x(t - \tau))' + q(t)H(x(t - \sigma)) = 0$$

are studied under various ranges of $p(t)$. Sufficient conditions are obtained for existence of bounded positive solutions of (E_1) .

Mathematics subject classification (2010): 34C10, 34C15, 34K40.

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