

## EXISTENCE OF POSITIVE SOLUTION AND NEW OSCILLATION CRITERIA FOR NONLINEAR FIRST-ORDER NEUTRAL DELAY DIFFERENTIAL EQUATIONS

SHYAM SUNDAR SANTRA

*Abstract.* In this work, oscillatory behaviour of the solutions of a class of nonlinear first-order neutral delay differential equations of the form

$$(E_1) \quad (x(t) + p(t)x(t - \tau))' + q(t)H(x(t - \sigma)) = f(t)$$

and

$$(E_2) \quad (x(t) + p(t)x(t - \tau))' + q(t)H(x(t - \sigma)) = 0$$

are studied under various ranges of  $p(t)$ . Sufficient conditions are obtained for existence of bounded positive solutions of  $(E_1)$ .

*Mathematics subject classification* (2010): 34C10, 34C15, 34K40.

*Keywords and phrases:* oscillation, nonoscillation, non-linear, delay, neutral differential equations, asymptotic behaviour, existence of positive solution, Knaster-Tarski fixed point theorem, Banach fixed point theorem.

### REFERENCES

- [1] F. N. Ahmed, R. R. Ahmad, U. K. S. Din and M. S. M. Noorani, *Oscillation criteria of first order neutral delay differential equations with Variable Coefficients*, Abst. Appl. Anal., Vol. 2013, Article ID 489804, <http://dx.doi.org/10.1155/2013/489804>
- [2] F. N. Ahmed, R. R. Ahmad, U. K. S. Din and Mohd. S. M. Noorani, *Oscillations for Nonlinear Neutral Delay Differential Equations with Variable Coefficients*, Abstract and Applied Analysis., Volume 2014, Article ID 179195, <http://dx.doi.org/10.1155/2014/179195>.
- [3] F. N. Ahmed, R. R. Ahmad, U. K. S. Din and M. S. M. Noorani, *Oscillation criteria for nonlinear functional differential equations of neutral type*, J. Inequ. Appl., DOI 10.1186/s13660-015-0608-5 (2015), 2015:97.
- [4] L. Berezansky and E. Braverman, *Oscillation criteria for a linear neutral differential equation*, J. Math. Anal. Appl. 286 (2003), 601–617.
- [5] P. Das and N. Misra, *A necessary and sufficient condition for the solutions of a functional differential equation to be oscillatory or tend to zero*, J. Math. Anal. Appl., 204(1997), pp. 78–87.
- [6] L. H. Erbe, Q. Kong and B. G. Zhang, *Oscillation Theory for Functional-Differential Equations*. Marcel Dekker, Inc., New York, (1995).
- [7] E. M. Elabbasy, T. S. Hassan and S. H. Saker, *Oscillation criteria for first-order nonlinear neutral delay differential equations*, Electronic Journal of Differential Equations., Vol. 2005 (2005), No. 134, pp. 1–18.
- [8] I. Gyori and G. Ladas, *Oscillation Theory of Delay Differential Equations with Applications*, Oxford University press., (1991).
- [9] J.R. Graef, R. Savithri and E. Thandapani, *Oscillation of First Order Neutral Delay Differential Equations*, Electronic journal of qualitative theory of differential equations., Proc. 7th Coll. QTDE, (2004), No. 12 1–11.
- [10] J.K. Hale, *Theory of Functional Differential Equations*, Springer, New York(1977).

- [11] I. Kubiacyk and S.H. Saker, *Oscillation of solutions to neutral delay differential equations*, Math. Slovaca., 52 (2002), No. 3, pp. 343–359.
- [12] I. Kubiacyk, S.H. Saker and J. Morchalo, *New oscillation criteria for first order nonlinear neutral delay differential equations*, Appl. Math. Compu., 142 (2003), 225–242.
- [13] B. Karpuz and O. Ocalan, *Oscillation criteria for some classes of linear delay differential equations of first-order*, Bulletin of the Institute of Mathematics, Academia Sinica (New Series), Vol. 3 (2008), No. 2, pp. 293–314.
- [14] B. Liu and L. Huang, *Existence and uniqueness of periodic solutions for a kind of first order neutral functional differential equations*, J. Math. Anal. Appl., 322 (2006), 121–132.
- [15] Z. Liu, S.M. Kangb and J.S. Ume, *Existence of bounded nonoscillatory solutions of first-order nonlinear neutral delay differential equations*, Computers and Mathematics with Applications., 59 (2010), 3535–3547.
- [16] G. Liu and J. Yan, *Global asymptotic stability of nonlinear neutral differential equation*, Commun Nonlinear Sci Numer Simulat., 19 (2014) 1035–1041.
- [17] H.A. El-Morshedy, *On the distribution of zeros of solutions of first order delay differential equations*, Nonlinear Analysis: Theory, Methods and Applications., 74 (2011), 3353–3362.
- [18] N. Parhi and R.N. Rath, *On oscillation and asymptotic behaviour of solutions of forced first order neutral differential equations*, Indian Acad. Sci. (Math. Sci.), Vol. III, No. 3, Aug (2001), pp. 337–350.
- [19] N. Parhi and R.N. Rath, *Oscillation criteria for forced first order neutral differential equations with variable coefficients*, J. Math. Anal. Appl., 256 (2001), 525–541.
- [20] X. H. Tang, *Oscillation for first-order nonlinear delay differential equations*, J. Math. Anal. Appl., 264 (2001), 510–521.
- [21] X.H. Tang and J.S. Yu, *Linearized oscillation of first-order nonlinear neutral delay differential equations*, J. Math. Anal. Appl., 258 (2001), 194–208.
- [22] S. Tanaka, *Oscillation of solutions of first-order neutral differential equations*, Hiroshima Math. J. 32 (2002), 79–85.
- [23] X. H. Tang and X. Lin, *Necessary and sufficient conditions for oscillation of first order nonlinear neutral differential equations*, J. Math. Anal. Appl., 321 (2006), 553–568.
- [24] X. Zhang and J. Yan, *Oscillation criteria for first order neutral differential equations with positive and negative coefficients*, J. Math. Anal. Appl., 253 (2001), 204–214.
- [25] W.P. Zhang, W. Feng, J.R. Yan and J.S. Song, *Existence of Nonoscillatory Solutions of First-Order Linear Neutral Delay Differential Equations*, Computers and Mathematics with Applications, 49 (2005), 1021–1027.