Differential
Equations
& Applications
Volume 8, Number 1 (2016), 69–76
doi:10.7153/dea-08-05

INEQUALITIES FOR ZEROS OF SOLUTIONS TO SECOND ORDER ODE WITH ONE SINGULAR POINT

MICHAEL GIL'

Abstract. We consider the equation $y''+P(z)y'+Q(z)y=0$ $(z\in\mathbb{C})$, where

$$P(z)=\sum_{k=0}^{n_P}p_kz^{k-1} \text{ and } Q(z)=\sum_{k=0}^{n_Q}q_kz^{k-2}$$

with real coefficients p_k,q_j $(k=0,...,n_P;\ j=0,...,n_Q;n_P,n_Q<\infty)$.

Let $z_k(y),k=1,2,...$ be the nontrivial zeros of a solution $y(z)$ to that equation. Estimates for the sums $\sum_{k=1}^{j}\frac{1}{|z_k(y)|}$ $(j=1,2,...)$ are derived. Applications of the obtained estimates to the counting function of the zeros of solutions are also discussed.

Mathematics subject classification (2010): 34C10, 34A30.

Keywords and phrases: ODE with a singular point, complex zeros of solutions, counting function, zero free domain.

REFERENCES

[1] N. ANGHEL, *Stieltjes-Calogero-Gil relations associated to entire functions of finite order*, J. of Mathem. Phys., **51**, no. 5 (2010) 251–262.

[2] B. BELAIDI, *Oscillation of fast growing solutions of linear differential equations in the unit disc*, Acta Univ. Sapientiae Mathematica, **2**, no. 1 (2010), 25–38.

[3] T. B. CAO AND H. X. YI, *On the complex oscillation theory of linear differential equations with analytic coefficients in the unit disc*, Acta Math. Sci., **28A (6)** (2008), 1046–1057.

[4] T. B. CAO, LIU KAI AND XU HONG-YAN, *Bounds for the sums of zeros of solutions of $u(m)=P(z)u$ where P is a polynomial*. Electron. J. Qual. Theory Differ. Equ. **no. 60**, (2011), 10 pp.

[5] YU L. DALECKII AND M.G. KREIN, *Stability of Solutions of Differential Equations in Banach Space*, Amer. Math. Soc., Providence, R. I. 1971.

[6] M.I. GIL', *Localization and Perturbation of Zeros of Entire Functions*, CRC Press, Taylor and Francis Group, New York, 2010.

[7] M.I. GIL', *Bounds for zeros of solutions of second order differential equations with polynomial coefficients*, Results Math., **59** (2011), 115–124.

[8] M.I. GIL', *Perturbation of zeros of solutions to second order differential equations with polynomial coefficients*, Acta Matematica Scientia, **32 (3)**, (2012), 1083–1092.

[9] M.I. GIL', *Sums of zeros of solutions to second order ODE with non-polynomial coefficients*, Electron. J. Diff. Equ., **Vol. 2012** (2012), no. 107, 1–8.

[10] M.I. GIL', *Sums of zeros of solutions to non-homogeneous ODE with polynomial coefficients*, J. Math. Anal. Appl. **421**, no. 2, (2015), 1917–1924.

[11] E. HILLE, *Lectures on Ordinary Differntial Equations*, Addison Wesley Publiching Company, Ontario, 1969.

[12] E.L. INCE, *Ordinary Differential Equations*, Dover Publ., New York, 1978.

[13] I. LAINE, *Nevanlinna Theory and Complex Differential Equations*, Walter de Gruyter Berlin, 1993

[14] Z. LATREUCH, B. BELAIDI AND A. EL FARISSI, *Complex oscillation of differential polynomials in the unit disc*, Periodica Mathematica Hungarica, **66 (1)**, (2013), 45–60.

Paper DEA-08-05

[15] G.M. MUMINOV, *On the zeros of solutions of the differential equation* $\omega^{(2m)} + p(z)\omega = 0$. Demonstr. Math., **35**, no. 1 (2002), 41–48.
[16] F. PENG AND Z. X. CHEN, *On the growth of solutions of some second-order differential equations*, J. Ineq. Appl., **2011** (2011), 1–9.
[17] J. TU AND Z. X. CHEN, *Zeros of solutions of certain second order linear differential equation*, J. Math. Anal. Appl. **332**, no. 1 (2007), 279–291.
[18] J. F. XU AND H. X. YI, *Solutions of higher order linear differential equations in an angle*, Appl. Math. Letters, **22**, no. 4 (2009), 484–489.