

GLOBAL DYNAMICS OF A DELAYED DIFFUSIVE TWO-STRAIN DISEASE MODEL

DANXIA CHEN AND ZHITING XU

Abstract. The aim of this paper is to investigate the global dynamics of a delayed diffusive two-strain disease model. We first study the well-posedness of the model. And then, by selecting appropriate Lyapunov functionals, we demonstrate that the global stability of the model is fully determined by the basic reproduction number. Furthermore, using Schauder fixed point theorem and constructing a pair of upper-lower solutions, we show that the model admits a traveling wave solution connecting the disease free and co-existence equilibria.

Mathematics subject classification (2010): 34K20, 35C07, 35K57, 92D30.

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