Differential
Equations
& Applications
Volume 8, Number 2 (2016), 123–134
doi:10.7153/dea-08-08

EXPONENTIAL DICHOTOMY OF LINEAR AUTONOMOUS SYSTEMS OVER TIME SCALES

AKBAR ZADA, TONGXING LI, SAMREEN ISMAIL AND OMAR SHAH

Abstract. In this paper we study the exponential stability and exponential dichotomy of the first order linear dynamic equation $z^{\Delta}(s) = Mz(s)$ in terms of the boundedness of solutions of the following Cauchy problems:

$$\begin{cases} z^{\Delta}(s) = Mz(s) + f(s)Qb, \ 0 \leqslant s \in \mathbb{T}, \\ z(0) = 0 \end{cases}$$

and

$$\begin{cases} w^{\Delta}(s) = -Mw^{\sigma} + f(s)(I-Q)b, \\ w(0) = 0, \end{cases}$$

where $\mathbb{T}$ is a time scale, M is a regressive matrix, b is a non-zero vector in $\mathbb{C}^m$, $f(s)$ is a bounded and right-dense continuous function on $\mathbb{T}$, and Q is a projection on $\mathbb{C}^m$.

Mathematics subject classification (2010): 34D09, 34N05, 34D05.

Keywords and phrases: exponential stability, Cauchy problem, first order linear dynamic equation, time scale.

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