

EXPONENTIAL DICHOTOMY OF LINEAR AUTONOMOUS SYSTEMS OVER TIME SCALES

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Abstract. In this paper we study the exponential stability and exponential dichotomy of the first order linear dynamic equation $z^\Delta(s) = Mz(s)$ in terms of the boundedness of solutions of the following Cauchy problems:

$$\begin{cases} z^\Delta(s) = Mz(s) + f(s)Qb, & 0 \leq s \in \mathbb{T}, \\ z(0) = 0 \end{cases}$$

and

$$\begin{cases} w^\Delta(s) = -Mw^\sigma + f(s)(I - Q)b, \\ w(0) = 0, \end{cases}$$

where \mathbb{T} is a time scale, M is a regressive matrix, b is a non-zero vector in \mathbb{C}^m , $f(s)$ is a bounded and right-dense continuous function on \mathbb{T} , and Q is a projection on \mathbb{C}^m .

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