

A SURVEY ON THE EXISTENCE, UNIQUENESS AND REGULARITY QUESTIONS TO FULLY NONLINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. The aim of this survey paper is to provide the recent developments on the existence, uniqueness and regularity results to fully nonlinear elliptic equations of the form

$$\begin{cases} F(x, u, Du, D^2 u) = f(x) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in \mathbb{R}^n .

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