OSCILLATORY AND ASYMPTOTIC BEHAVIOUR OF SECOND ORDER NEUTRAL DYNAMIC EQUATIONS WITH POSITIVE AND NEGATIVE COEFFICIENTS

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Abstract. In this paper, oscillatory and asymptotic properties of solutions of nonlinear second order neutral dynamic equations of the form

$$\left(r(t)\left(y(t) + p(t)y(\alpha(t)) \right)^{\Delta} \right)^{\Delta} + q(t)G(y(\beta(t))) - h(t)H(y(\gamma(t))) = 0$$

and

$$\left(r(t)\left(y(t) + p(t)y(\alpha(t)) \right)^{\Delta} \right)^{\Delta} + q(t)G(y(\beta(t))) - h(t)H(y(\gamma(t))) = f(t)$$

are studied under assumptions

$$\int\limits_{0}^{\infty} \frac{1}{r(t)} \Delta t < \infty \quad \text{and} \quad \int\limits_{0}^{\infty} \frac{1}{r(t)} \Delta t = \infty$$

for various ranges of $p(t)$, where $\mathbb{T}$ is a time scale with $\sup \mathbb{T} = \infty$, $t \in [t_0, \infty)_{\mathbb{T}}$, and $t_0 \geqslant 0$. Examples illustrating the results are included.

Mathematics subject classification (2010): 34C10, 34C15, 34N05.
Keywords and phrases: functional dynamic equations, oscillatory, neutral, time scale, nonlinear, second order, positive and negative coefficients.

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