

## OSCILLATORY AND ASYMPTOTIC BEHAVIOUR OF SECOND ORDER NEUTRAL DYNAMIC EQUATIONS WITH POSITIVE AND NEGATIVE COEFFICIENTS

SAROJ PANIGRAHI AND ERCAN TUNÇ

*Abstract.* In this paper, oscillatory and asymptotic properties of solutions of nonlinear second order neutral dynamic equations of the form

$$\left( r(t) (y(t) + p(t)y(\alpha(t)))^\Delta \right)^\Delta + q(t)G(y(\beta(t))) - h(t)H(y(\gamma(t))) = 0$$

and

$$\left( r(t) (y(t) + p(t)y(\alpha(t)))^\Delta \right)^\Delta + q(t)G(y(\beta(t))) - h(t)H(y(\gamma(t))) = f(t)$$

are studied under assumptions

$$\int_0^\infty \frac{1}{r(t)} \Delta t < \infty \quad \text{and} \quad \int_0^\infty \frac{1}{r(t)} \Delta t = \infty$$

for various ranges of  $p(t)$ , where  $\mathbb{T}$  is a time scale with  $\sup \mathbb{T} = \infty$ ,  $t \in [t_0, \infty)_{\mathbb{T}}$ , and  $t_0 \geq 0$ . Examples illustrating the results are included.

*Mathematics subject classification* (2010): 34C10, 34C15, 34N05.

*Keywords and phrases:* functional dynamic equations, oscillatory, neutral, time scale, nonlinear, second order, positive and negative coefficients.

### REFERENCES

- [1] R. P. Agarwal, M. Bohner, T. Li, C. Zhang, *A new approach in the study of oscillatory behaviour of even-order neutral delay differential equations*, Appl. Math. Comput. **225** (2013), 787–794.
- [2] D. K. Anderson and A. Zafer, *Nonlinear oscillation of second order dynamic equations on time scales*, Appl. Math. Lett. **22** (2009), 1591–1597.
- [3] D. D. Bainov, D.P. Mishev, *Oscillation Theory for Neutral Differential Equation with Delay*, Hilger, Bristol, 1991.
- [4] M. Bohner, *Dynamic Equations on Time Scales*, Birkhäuser, 2001.
- [5] M. Bohner, A. Peterson, *Advances in Dynamic Equations on Time Scale*, Birkhäuser, 2002.
- [6] L.H. Erbe, Q. Kong, B.G. Zhang, *Oscillation Theory for Functional Differential Equation*, Dekker, NewYork, 1995.
- [7] I. Gyori, G. Ladas, *Oscillation Theory for Delay Differential Equations with Applications*, Oxford Univ. Press, London, 1991.
- [8] S. Hilger, *Analysis on measure chains: a unified approach to continuous and discrete calculus*, Results Math. **18** (1990), 18–56.
- [9] B. Karpuz, J. V. Manojlovic, O. Ocalan, Y. Shoukaku, *Oscillation criteria for a class of second-order neutral delay differential equations*, Appl. Math. Comput. **210** (2009), 303–312.
- [10] I. Kubiacyzyk, S. H. Saker, A. Sikorska-Nowak, *Oscillation criteria for nonlinear neutral functional dynamic equations on time scales*, Math. Slovaca. **63** (2013), 263–290.
- [11] T. Li, Y. V. Rogovvchenko, *Oscillation theorems for second-order nonlinear neutral delay differential equations*, Abst. Appl. Anal. 2014 (2014), Article ID 594190, 5pages.

- [12] S. Panigrahi and P.R. Reddy, *On oscillatory fourth order nonlinear neutral delay dynamic equations*, *Comput. Math. Appl.* **62** (2011), 4258–4271.
- [13] H. Qin, N. Shang, Y. Lu, *A note on oscillation criteria of second order nonlinear neutral delay differential equations*, *Comput. Math. Appl.* **56** (2008), 2987–2992.
- [14] S. H. Saker and D. O'Regan, *New oscillation criteria for second-order neutral dynamic equations on time scales via Riccati substitution*, *Hiroshima Math. J.* **42** (1) (2012), 109–122.
- [15] E. Thandapani, V. Piramanantham and S. Pinelas, *Oscillation criteria for second-order neutral delay dynamic equations with mixed nonlinearities*, *Advances in Difference Equations*, **2011**, Article ID594190, 5pages.
- [16] E. Thandapani and V. Piramanantham, *Oscillation criteria for second order nonlinear neutral dynamic equations on time scales*, *Tamkang J. Math.* **43** (2012), 109–122.
- [17] A. K. Tripathy, *Oscillation properties of a class of neutral differential equations with positive and negative coefficients*, *Fasciculi Mathematici*, **45** (2010), 133–155.
- [18] A. K. Tripathy, *Oscillatory behaviour of a class of nonlinear second order mixed difference equations*, *Elect. J. Qual. Theory of Diff. Eqs.* **48** (2010), 1–19.
- [19] Q. Yang, Z. Xu, *Oscillation criteria for second order quasilinear neutral delay differential equations on time scales*, *Comput. Math. Appl.* **62** (2011), 3682–3691.
- [20] Shao-Yan Zhang and Qi-Ru Wang, *Oscillation criteria for second-order nonlinear dynamic equations on time scales*, *Abstract and Applied Analysis*, **2012** (2012), Article ID 743469, 20 pages.