

## EXISTENCE AND STABILITY RESULTS FOR NONLINEAR IMPLICIT FRACTIONAL DIFFERENTIAL EQUATIONS WITH DELAY AND IMPULSES

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*Abstract.* In this paper, we establish the existence, uniqueness and Ulam stability of solutions for a class of problem for nonlinear implicit fractional differential equations with impulse and Caputo fractional derivative. The arguments are based upon the Banach contraction principle, and Schaefer's fixed point theorem. We present two examples to show the applicability of our results.

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