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EXISTENCE AND STABILITY RESULTS FOR NONLINEAR IMPLICIT FRACTIONAL DIFFERENTIAL EQUATIONS WITH DELAY AND IMPULSES

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Abstract. In this paper, we establish the existence, uniqueness and Ulam stability of solutions for a class of problem for nonlinear implicit fractional differential equations with impulse and Caputo fractional derivative. The arguments are based upon the Banach contraction principle, and Schaefer's fixed point theorem. We present two examples to show the applicability of our results.

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REFERENCES

[1] S. Abbas, M. Benchohra and G.M. N'Guérékata, *Topics in Fractional Differential Equations*, Springer-Verlag, New York, 2012.
[2] S. Abbas, M. Benchohra and G.M. N'Guérékata, *Advanced Fractional Differential and Integral Equations*, Nova Science Publishers, New York, 2015.
[3] R.P. Agarwal and B. Ahmad, Existence theory for anti-periodic boundary value problems of fractional differential equations and inclusions, *Comput. Math. Appl.*, **62** (2011), 1200–1214.
[4] B. Ahmad and J.J. Nieto, Existence of solutions for impulsive anti-periodic boundary value problems of fractional order, *Taiwaness J. Math.*, **15** (3) (2011), 981–993.
[5] D.D. Bainov and S.G. Hristova, Integral inequalities of Gronwall type for piecewise continuous functions, *J. Appl. Math. Stoc. Anal.*, **10** (1997), 89–94.
[6] D. Baleanu, K. Diethelm, E. Scalas, and J.J. Trujillo *Fractional Calculus Models and Numerical Methods*, World Scientific Publishing, New York, 2012.
[7] D. Baleanu, Z.B. Güvenç, and J.A.T. Machado *New Trends in Nanotechnology and Fractional Calculus Applications*, Springer, New York, 2010.
[8] M. Benchohra, F. Berhoun, N. Hamidi and J.J. Nieto, Fractional differential inclusions with anti-periodic boundary conditions, *Nonlinear Anal. Forum*, **19** (2014), 27–35.
[9] M. Benchohra, N. Hamidi and J. Henderson, Fractional differential equations with anti-periodic boundary conditions, *Numer. Funct. Anal. Optim.* **34** (4) (2013), 404–414.
[10] M. Benchohra, J. Henderson and S. K. Ntouyas, *Impulsive Differential Equations and Inclusions*, Hindawi Publishing Corporation, Vol. 2, New York, 2006.
[11] M. Benchohra and J. E. Lazreg, Nonlinear fractional implicit differential equations, *Commun. Appl. Anal.*, **17** (2013), 471–482.
[12] M. Benchohra and J. E. Lazreg, Existence and uniqueness results for nonlinear implicit fractional differential equations with boundary conditions, *Rom. J. Math. Comput. Sc.*, **4** (2014), 60–72.
[13] Y. Chang, A. Anguraj and P. Karthikeyan, Existence results for initial value problems with integral condition for impulsive fractional differential equations, *J. Fract. Calc. Appl.*, **2** (7) (2012), 1–10.
[14] A. Chen and Y. Chen, Existence of solutions to anti-periodic boundary value problem for nonlinear fractional differential equations with impulses. *Adv. Difference. Equ.*, (2011), 17 pp.
[15] K. Diethelm, *The Analysis of Fractional Differential Equations*, Lecture Notes in Mathematics, 2010.

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[16] J. R. Graef, J. Henderson and A. Ouahab, *Impulsive Differential Inclusions. A Fixed Point Approch*, De Gruyter, Berlin/Boston, 2013.
[17] A. Granas and J. Dugundji, *Fixed Point Theory*, Springer-Verlag, New York, 2003.
[18] J. Hale and S.M. Verduyn Lunel, *Introduction to Functional Differential Equations*, , Applied Mathematicals Sciences, 99, Springer-Verlag, New York, 1993.
[19] J. Henderson and A. Ouahab, Impulsive differential inclusions with fractional order, *Comput. Math. Appl.*, **59** (2010), 1191–1226.
[20] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.
[21] A.A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, 204. Elsevier Science B.V., Amsterdam, 2006.
[22] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, *Theory of Impulsive Differntial Equations*, Worlds Scientific,Singapore, 1989.
[23] N. A. Perestyuk, V. A. Plotnikov, A. M. Samoilenko, N. V. Skripnik, *Differential Equation with Impulse Effects, Multivalued Right-hand Sides with Discontinuities* Walter de Gruyter, Berlin/Boston, 2011.
[24] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
[25] A.M. Samoilenko and N.A. Perestyuk, *Impulsive Differential Equations* World Scientific, Singapore, 1995.
[26] V.E. Tarasov, *Fractional Dynamics: Application of Fractional Calculus to Dynamics of particles, Fields and Media*, Springer, Heidelberg; Higher Education Press, Beijing, 2010.
[27] G. Wang, B. Ahmad and L. Zhang, Impulsive anti-periodic boundary value problem for nonlinear differential equations of fractional order, *Nonlinear Anal.*, **74** (2011), 792–804.
[28] G. Wang, B. Ahmad and L. Zhang, Some existence results for impulsive nonlinear fractional differential equations with mixed boundary conditions, *Comput. Math. Appl.* **62** (2011), 1389–1397.
[29] J. Wang, M. Feckan and Y. Zhou, Ulam's type stability of impulsive ordinary differential equations, *J. Math. Anal. Appl.*, **395** (2012), 258–264.
[30] H.Ye, J. Gao and Y. Ding, A generalized Gronwall inequality and its application to a fractional differential equation, *J. Math. Anal. Appl.*, **328** (2007), 1075–1081.
[31] L. Zhang and G. Wang, Existence of solutions for nonlinear fractional differential equations with impulses and anti-Periodic boundary conditions, *Electron. J. Qual. Theory Differ. Equ.*, 2011, No. 7, pp. 1–11.