CRITICAL GROWTH PROBLEMS FOR $\frac{1}{2}$–LAPLACIAN IN $\mathbb{R}$

J. GIACOMONI, P.K. MISHRA AND K. SREENADH

Abstract. We study the existence of weak solutions for fractional elliptic equations of the type,

$$(-\Delta)^{\frac{1}{2}}u + V(x)u = h(u), \quad u > 0 \text{ in } \mathbb{R},$$

where $h$ is a real valued function that behaves like $e^{u^2}$ as $u \to \infty$ and $V(x)$ is a positive, continuous unbounded function. Here $(-\Delta)^{\frac{1}{2}}$ is the fractional Laplacian operator. We show the existence of mountain-pass solution when the nonlinearity is superlinear near $t = 0$. We also study the corresponding critical exponent problem for the Kirchhoff equation

$$m\left(\int_{\mathbb{R}}|(-\Delta)^{\frac{1}{2}}u|^2\,dx + \int_{\mathbb{R}}V(x)u^2\,dx\right)((-\Delta)^{\frac{1}{2}}u + V(x)u) = f(u) \text{ in } \mathbb{R},$$

where $f(u)$ behaves like $e^{u^2}$ as $u \to \infty$ and $f(u) \sim u^\theta$, with $\theta > 3$, as $u \to 0$.

Keywords and phrases: Trudinger-Moser inequality, square root of Laplacian, Kirchhoff.

REFERENCES


