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CRITICAL GROWTH PROBLEMS FOR $\frac{1}{2}$–LAPLACIAN IN $\mathbb{R}$

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Abstract. We study the existence of weak solutions for fractional elliptic equations of the type,

$$(-\Delta)^{\frac{1}{2}}u+V(x)u=h(u),\ u>0 \text{ in } \mathbb{R},$$

where h is a real valued function that behaves like e^{u^2} as $u\to\infty$ and $V(x)$ is a positive, continuous unbounded function. Here $(-\Delta)^{\frac{1}{2}}$ is the fractional Laplacian operator. We show the existence of mountain-pass solution when the nonlinearity is superlinear near $t=0$. We also study the corresponding critical exponent problem for the Kirchhoff equation

$$m\left(\int_{\mathbb{R}}|(-\Delta)^{\frac{1}{4}}u|^2dx+\int_{\mathbb{R}}V(x)u^2dx\right)\left((-\Delta)^{\frac{1}{2}}u+V(x)u\right)=f(u) \text{ in } \mathbb{R},$$

where $f(u)$ behaves like e^{u^2} as $u\to\infty$ and $f(u)\sim u^{\theta}$, with $\theta>3$, as $u\to 0$.

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