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ON NONLINEAR PERTURBATIONS OF STURM–LIOUVILLE PROBLEMS IN DISCRETE AND CONTINUOUS SETTINGS

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Abstract. In this paper we provide sufficient conditions for the existence of solutions to certain classes of second-order discrete and continuous systems. In particular, we examine problems that can be posed as nonlinear perturbations of Sturm-Liouville problems. We first provide a lemma on the invertibility of a nonlinearly-perturbed invertible linear operator, and apply this result to extend previous work on these topics.

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