

SOLOW MULTI-CAPITAL GROWTH MODEL DESCRIBED BY A SYSTEM OF DIFFERENTIAL EQUATIONS ON TIME SCALES

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Abstract. In this paper we derive a system of differential equations on time scales of the Solow type corresponding to a production function depending on several capitals. A sufficient condition for the exponential stability of the steady-state solution with positive coordinates is proved. The obtained results are applied to the case of the Cobb-Douglas type production function.

Mathematics subject classification (2010): 34N05, 26E70, 97E40, 97M10.

Keywords and phrases: Δ -derivative, time scales, multi-capital, stability, Cobb-Douglas function.

REFERENCES

- [1] R. AGARWAL, M. BOHNER, D. O' REGAN AND A. PETERSON, *Dynamic Equations on time scales: A survey*, J. Math. Comp. and Appl. Math., **141** (2002), 1–26.
- [2] E. ACCINELLI AND J. G. BRIDA, *Population growth and the Solow-Swan model*, International Journal of Ecological Economics and Statistics, **8**, 7 (2007), 54–63.
- [3] M. BOHNER AND A. PETERSON, *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, MA 2003.
- [4] M. BOHNER AND A. PETERSON, *Dynamic Equations on Time Scales: An Introduction with Applications*, Birkhäuser, Boston, Basel, Berlin 2001.
- [5] M. BOHNER, J. HEIM AND A. LIU, *Solow models on time scales*, CUBO, A Mathematica Journal **15**, 1 (2013), 13–32.
- [6] M. BOHNER, J. HEIM AND A. LIU, *Qualitative analysis of a Solow model on time scales*, J. Concr. Appl. Math. **13**, 3–4 (2015), 183–197.
- [7] E. BRESTOVANSKÁ AND M. MEDVEĎ, *Solow differential equations on time scales: A unified approach to continuous and discrete Solow growth model*, Differ. Equ. Appl., **5**, 4 (2013), 473–488.
- [8] S. BRIANZONI, C. MAMMANA AND E. MICHETTI, *Local and global dynamics in discrete time growth model with nonconcave production function*, Discrete Dyn. Nat. Soc., (2012), Article ID 536570, 22 pages, doc:10.1155/2012/53657.
- [9] S. BRIANZONI, C. MAMMANA AND E. MICHETTI, *Complex dynamics in the neoclassical growth model with differential saving and non-constant labour force growth*, Stud. Nonlinear Dyn. Econom., **11**, 3 (2007).
- [10] S. BRIANZONI, C. MAMMANA AND E. MICHETTI, *Global attractor in Solow growth model with differential savings and endogenous labour force growth*, AMS Periodicals, Modelling Measurement and Control D **29**, 2 (2008), 19–37.
- [11] J. G. BRIDA AND E. L. MALDONADO, *Closed form solutions to a generalization of the Solow growth model*, Applied Mathematical Sciences, **1**, 40 (2007), 1991–2000.
- [12] J. G. BRIDA AND J. S. PEREYRA, *The Solow model in discrete time and decreasing population growth rate*, Economic Bulletin, **3**, 41 (2008), 1–14.
- [13] D. CHEBAN, C. MAMMANA AND E. MICHETTI, *Global attractors of quasi-linear non-autonomous difference equations: A growth model with endogenous population growth*, Nonlinear Anal.: Serie B, **14** (2013), 1716–1731.
- [14] J. HEIM, *Economics and Finance on time scales*, Dissertation, Faculty of the Graduate School, Missouri University of Science and Technology, 2012 (Advisor: M. Bohner).

- [15] S. HILGER, *Ein Maßkettenkalkül Anwendung auf Zentrumsmannigfaltigkeiten*, PhD thesis, Universität Würzburg, 1988.
- [16] S. HILGER, *Analysis on measure chains – A unified approach to continuous and discrete calculus*, Results Math., **18** (1990), 18–56.
- [17] S. HILGER, *Differential and difference calculus – unified!*, Nonlinear Analysis, TMA **30**, 5 (1997), 2683–2694.
- [18] J. HOFFACKER AND B. JAKSON, *Stability results for higher dimensional equations on time scales*, Int. J. Dynamical Systems and Differential equations, **3**, No. 1–2 (2011), 48–58.
- [19] T. KULIK AND C. C. TISDELL, *Volterra integral equations on time scales, Basic qualitative and quantitative results with applications to initial value problems on unbounded domains*, International Journal of Difference Equations, **3**, 1 (2008), 103–133.
- [20] M. G. MANKIW, D. ROMER AND D. N. WEIL, *A contribution to the empirics of economic growth*, The Quarterly J. of Economics, **107**, 2 (1992), 407–437.
- [21] V. LAKSHMIKANTAN, S. SIVASUNDRAM, B. KAYAMKALAN, *Dynamic Systems on Measure Chains*, Kluwer Academic Publishers, Netherlands 1996.
- [22] M. MEDVEĎ, *Fundamentals of Dynamical Systems and Bifurcation Theory*, Adam Hilger, Bristol, Philadelphia and New York 1992.
- [23] R. M. SOLOW, *Contribution to the theory of economic growth*, Quarterly Journal of Economics, **70**, 1 (1956), 65–94.
- [24] R. M. SOLOW, *Technical change and the aggregate production function*, Review of Economics and statistics, **39**, 3 (1957), 312–320.
- [25] T. W. SWAN, *Economic growth and capital accumulation*, Economic Record, **32**, 2 (1956), 334–361.
- [26] C. C. TISDELL AND A. ZAIDI, *Basic qualitative and quantitative results for solutions to nonlinear, dynamic equations on time scales*, Nonlinear Anal., **68**, 11 (2008), 3504–3524.
- [27] C. C. TISDELL AND A. H. ZAIDI, *Successive approximations to solutions of dynamic dynamic equations on time scales*, Comm. Appl. Nonlinear Anal., **16**, 1 (2009), 61–87.
- [28] P. VANHOUDT, *Are public and private outlays for physical and knowledge capital equally productive?* Applied Economics, **31** (1999), 1401–1410.
- [29] J. VANHOUDT, *How productive are capital investments in Europe?* European Investment Bank (EIB). ISSN 0257-7755, **5**, 2 81–106 (<http://hdl.handle.net/10419/44797>).
- [30] H. WIESE, *Intermediate Growth Theory*, Springer-Verlag, Berlin, Heidelberg, New York 2006.