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SECOND–ORDER FUNCTIONAL PROBLEMS WITH A RESONANCE OF DIMENSION ONE

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Abstract. We obtain, using the coincidence degree theory, solvability conditions for all possible resonance scenarios $Lu = u'' = f(t,u,u') = Nu$, with linear functional conditions $B_i u = 0$, $i = 1,2$ with $\dim\ker L = 1$. Our work generalizes and improves the results of Zhao and Liang [18] and Cui [3] in several directions. We also construct a meaningful example of a nonlinear functional problem for a pendulum equation which not only satisfies the assumptions of an existence theorem but also has a closed-form solution.

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