

## MULTIPLE SOLUTIONS TO THE NONHOMOGENEOUS KIRCHHOFF TYPE PROBLEM INVOLVING A NONLOCAL OPERATOR

WENJING CHEN AND JUNHUI XIE

**Abstract.** This paper examines the nonhomogenous Kirchhoff type equation that involves a non-local operator. Using Ekeland's variational principle and the Mountain pass theorem, the existence of multiple solutions is established.

*Mathematics subject classification (2010):* 35J20, 35J60, 47G20.

*Keywords and phrases:* Kirchhoff type problem, non-local integrodifferential operator, Ekeland's variational principle, Mountain pass theorem.

### REFERENCES

- [1] C.O. ALVES, F.J.S.A. CORRÊA, T.F. MA, *Positive solutions for a quasilinear elliptic equation of Kirchhoff type*, Comput. Math. Appl., **49**, (1) (2005), 85–93.
- [2] C. CHEN, J. HUANG, L. LIU, *Multiple solutions to the nonhomogeneous  $p$ -Kirchhoff elliptic equation with concave-convex nonlinearities*, Applied Mathematics Letters, **26**, (7) (2013), 754–759.
- [3] C. CHEN, Y. KUO, T. WU, *The Nehari manifold for a Kirchhoff type problem involving sign-changing weight functions*, J. Differential Equations, **250**, (4) (2011), 1876–1908.
- [4] B. CHENG, X. WU, *Existence results of positive solutions of Kirchhoff type problems*, Nonlinear Anal., **71**, (10) (2009), 4883–4892.
- [5] F.J.S.A. CORRÊA, G. M. FIGUEIREDO, *On a  $p$ -Kirchhoff equation via Krasnoselskii's genus*, Appl. Math. Lett., **22** (2009), 819–822.
- [6] E. DI NEZZA, G. PALATUCCI AND E. VALDINOCI, *Hitchhiker's guide to the fractional Sobolev spaces*, Bull. Sci. Math., **136** (2012), 521–573.
- [7] I. EKELAND, *On the variational principle*, J. Math. Anal. Appl., **47** (1974), 324–353.
- [8] A. FISCHELLA, E. VALDINOCI, *A critical Kirchhoff type problem involving a nonlocal operator*, Nonlinear Analysis, **94** (2014), 156–170.
- [9] X. HE, W. ZOU, *Infinitely many positive solutions of Kirchhoff type problems*, Nonlinear Anal. **70** (3)(2009), 1407–1414.
- [10] D. LIU, P. ZHAO, *Multiple nontrivial solutions to a  $p$ -Kirchhoff equation*, Nonlinear Anal., **75** (2012), 5032–5038.
- [11] K. PERERA, Z. T. ZHANG, *Nontrivial solutions of Kirchhoff-type problems via the Yang index*, J. Differential Equations, **221** (2006), no. 1, 246–255.
- [12] P.H. RABINOWITZ, *Minimax methods in critical point theory with applications to differential equations*, CBMS Regional Conference Series in Mathematics, vol. **65**, Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1986.
- [13] R. SERVADEI AND E. VALDINOCI, *Lewy-Stampacchia type estimates for variational inequalities driven by (non)local operators*, Rev. Mat. Iberoam., **29**, (3) (2013), 1091–1126.
- [14] R. SERVADEI AND E. VALDINOCI, *Mountain Pass solutions for non-local elliptic operators*, J. Math. Anal. Appl., **389** (2012), 887–898.
- [15] R. SERVADEI AND E. VALDINOCI, *Variational methods for non-local operators of elliptic type*, Discrete and Continuous Dynamical Systems, **5** (2013), 2105–2137.
- [16] J.J. SUN AND C.L. TANG, *Existence and multiplicity of solutions for Kirchhoff type equations*, Nonlinear Analysis, **74** (2011), 1212–1222.

- [17] Q. XIE, X. WU, C.L. TANG, *Existence and multiplicity of solutions for Kirchhoff type problem with critical exponent*, Commun. Pure Appl. Anal., **12**, (6) (2013), 2773–2786.
- [18] Z. ZHANG, K. PERERA, *Sign changing solutions of Kirchhoff type problems via invariant sets of descent flow*, J. Math. Anal. Appl., **317**, (2) (2006), 456–463.