Zeros’ Distribution of the First Kind Bessel Functions

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Abstract. The aim of this paper is to investigate the zeros’ distribution of the first kind Bessel functions $J_\nu(z)$ of order $\nu \geq 1$. The problem arises from the conjecture given by the work [8] which considered the existence of smooth solutions for one-dimensional compressible Euler equation with gravity. In this article we show that $J_\nu(L\theta) \neq 0$ for any integer $L \geq 2$ provided that $J_\nu(\theta) = 0$, $\nu \geq 1$ and $\theta$ is sufficiently large. Moreover, if $\nu$ is half of an odd integer, we can remove the restriction of large $\theta$ and show that $J_\nu(L\theta) \neq 0$ for any integer $L \geq 2$.

Keywords and phrases: Bessel function, Siegel’s theorem, Nash-Moser Theorem.

References