Differential
Equations
& Applications
Volume 8, Number 3 (2016), 377–384
doi:10.7153/dea-08-20

ZEROS' DISTRIBUTION OF THE FIRST KIND BESSEL FUNCTIONS

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Abstract. The aim of this paper is to investigate the zeros' distribution of the first kind Bessel functions $J_\nu(z)$ of order $\nu \geqslant 1$. The problem arises from the conjecture given by the work [8] which considered the existence of smooth solutions for one-dimensional compressible Euler equation with gravity. In this article we show that $J_\nu(L\theta) \neq 0$ for any integer $L \geqslant 2$ provided that $J_\nu(\theta) = 0$, $\nu \geqslant 1$ and θ is sufficiently large. Moreover, if ν is half of an odd integer, we can remove the restriction of large θ and show that $J_\nu(L\theta) \neq 0$ for any integer $L \geqslant 2$.

Mathematics subject classification (2010): 33C10, 34A99.
Keywords and phrases: Bessel function, Siegel's theorem, Nash-Moser Theorem.

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