

EXAMPLES OF SHOCKS IN POPULATION DYNAMICS

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Abstract. The aim of this paper is to study some shocks in population dynamics by means of the recent theories of generalized functions in order to have a better understanding of the long-term behavior of these phenomena. The shocks analyzed can be wars, genocides, epidemics, natural disasters, cancers... Population dynamics is represented by the transport equation and shocks by initial data which are distributions. This justifies to search for solutions in Colombeau algebra. Moreover we build well-posed problems. We study two models, the genesis model and a top hat condition.

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