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ON THE RAYLEIGH–PLATEAU INSTABILITY. THE REGULARITY IN H^3_{per}

A. Alriyabi

Abstract. In this paper, we study the Rayleigh-Plateau instability of a cylindrical pore. We are interested in the model developed by Spencer et al. [20], Kirill et al. [12] and Boutat et al. [3] in absence of the stress. We obtain a nonlinear parabolic PDE of fourth order. We obtain the local existence and uniqueness of the solution of this problem. The global existence of the solution and the convergence to the mean value of the initial data for long time, represent the main results of this work. In this study, we give also a numerical tests in order to validate the theoretical results.

Mathematics subject classification (2010): 26D15, 26A51, 32F99, 41A17.
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