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SPECTRAL ANALYSIS OF A NONLINEAR BOUNDARY–VALUE PROBLEM IN A PERFORATED DOMAIN. APPLICATIONS TO THE FRIEDRICHS INEQUALITY IN L_p

YULIA KOROLEVA

Abstract. The paper deals with asymptotic expansion for p-Laplace boundary-value problem in a domain periodically perforated along the boundary. It is assumed that the later boundary of the domain is subject to the Neumann boundary condition while the Dirichlet condition is set on the boundary of small sets. The asymptotic expansion for the first eigenelement is constructed. This result is applied to derive the asymptotics of the best constant in the Friedrichs inequality.

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