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COUPLED SYSTEMS OF FRACTIONAL ∇–DIFFERENCE BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we study the existence of solutions for a coupled system of two-point fractional ∇-difference boundary value problems of the form

$$\begin{pmatrix} \nabla^{\alpha}_{a^+} u(t) \\ \nabla^{\beta}_{a^+} v(t) \end{pmatrix} + \begin{pmatrix} f(t,v(t)) \\ g(t,u(t)) \end{pmatrix} = 0,$$

$$\begin{pmatrix} u(a+1) \\ u(b+1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v(a+1) \\ v(b+1) \end{pmatrix},$$

where $1 < \alpha, \beta \leqslant 2$, $t \in [a+2, b+1]_{\mathbb{N}} = \{a+2, a+3, ..., b, b+1\}$, $a, b \in \mathbb{Z}$ such that $a \geqslant 0, b \geqslant 3$ and the functions $f, g : [a+2, b+1]_{\mathbb{N}} \times \mathbb{R} \to \mathbb{R}$ are continuous. Our analysis relies on the Green functions and the nonlinear alternative of Leray-Schauder and Krasnoselśkii-Zabreiko fixed point theorems. At the end we give some numerical examples to illustrate the main results.

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