

COUPLED SYSTEMS OF FRACTIONAL ∇ -DIFFERENCE BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we study the existence of solutions for a coupled system of two-point fractional ∇ -difference boundary value problems of the form

$$\begin{aligned} \left(\begin{array}{c} \nabla_{a^+}^\alpha u(t) \\ \nabla_{a^+}^\beta v(t) \end{array} \right) + \left(\begin{array}{c} f(t, v(t)) \\ g(t, u(t)) \end{array} \right) &= 0, \\ \left(\begin{array}{c} u(a+1) \\ u(b+1) \end{array} \right) &= \left(\begin{array}{c} 0 \\ 0 \end{array} \right) = \left(\begin{array}{c} v(a+1) \\ v(b+1) \end{array} \right), \end{aligned}$$

where $1 < \alpha, \beta \leq 2$, $t \in [a+2, b+1]_{\mathbb{N}} = \{a+2, a+3, \dots, b, b+1\}$, $a, b \in \mathbb{Z}$ such that $a \geq 0, b \geq 3$ and the functions $f, g : [a+2, b+1]_{\mathbb{N}} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous. Our analysis relies on the Green functions and the nonlinear alternative of Leray-Schauder and Krasnoselskii-Zabreiko fixed point theorems. At the end we give some numerical examples to illustrate the main results.

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