

PATTERNS IN A BALANCED BISTABLE EQUATION WITH HETEROGENEOUS ENVIRONMENTS ON SURFACES OF REVOLUTION

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Abstract. We use the variational concept of Γ -convergence to obtain sufficient conditions that guarantee existence, stability and the geometric structure of four families of stationary solutions to the singularly perturbed parabolic equation $\partial_t u_\varepsilon = \varepsilon^2 \Delta u_\varepsilon + f(u_\varepsilon, x)$ on surfaces of revolution. We consider the bistable function $f(u, x) = -(u - a(x))(u - b(x))(u - c(x))$ and the conditions found relate the functions a, b, c to the geometry of the surface where such functions are defined.

Mathematics subject classification (2010): 35K57, 35B36, 35R01, 35B25, 35B35, 34K20, 58J32.

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