EXACT TRAVELING WAVES SOLUTIONS FOR
LONG WAVES AND BLOW–UP PHENOMENA

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Abstract. In this work we find exact traveling waves solutions to the fifth-order KDV-BBM type model that appear to describe the propagation of long waves in shallow water. We study the possibility of blow-up phenomenon of the fifth-order KDV-BBM type model under certain restrictions on the coefficients. Moreover, by applying the Ince transformation we also establish exact traveling waves solutions to the nonlinear evolution equation Benney-Lin type.

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