

## OSCILLATION OF SECOND ORDER NONLINEAR DIFFERENTIAL EQUATION WITH SUB-LINEAR NEUTRAL TERM

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*Abstract.* In this paper the authors established sufficient conditions for the oscillation of all solutions of a nonlinear differential equation

$$\left( a(t)(x(t) + p(t)x^\alpha(\tau(t)))' \right)' + q(t)x^\beta(\sigma(t)) = 0, \quad t \geq t_0,$$

where  $\alpha$  and  $\beta$  are ratio of odd positive integers. The results obtained here extend and improve some of the existing results. Examples are included to illustrate the importance of the results.

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