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NONLINEAR MODEL OF QUASI–STATIONARY PROCESS IN CRYSTALLINE SEMICONDUCTOR

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Abstract. We consider the question of global existence and asymptotics of small, smooth, and localized solutions of a certain pseudoparabolic equation in one dimension, posed on half-line $x > 0$,

$$\begin{cases} \left(1-\partial_x^2\right) u_t = \partial_x^2 \left(u + \alpha_2 \left(|u|^{q_2} u\right)\right) + \alpha_1 |u|^{q_1} u, \; x \in \mathbb{R}^+, \; t > 0, \\ u(0,x) = u_0(x), \, x \in \mathbb{R}^+, \\ u(0,t) = h(t), \end{cases} \tag{0.1}$$

where $\alpha_i \in \mathbb{R}, q_i > 0, i = 1,2, u \, : \, R_x^+ \times R_t^+ \in \mathbb{C}$. This model is motivated by the a wave equation for media with a strong spatial dispersion, which appear in the nonlinear theory of the quasy-stationary processes in the electric media. We show that the problem (0.1) admits global solutions whose long-time behavior depend on boundary data. More precisely, we prove global existence and modified by boundary scattering of solutions.

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Paper DEA-09-04

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