

SIMILARITY SOLUTIONS OF MIXED CONVECTION BOUNDARY-LAYER FLOWS IN A POROUS MEDIUM

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Abstract. The similarity differential equation $f''' + ff'' + \beta f'(f' - 1) = 0$ with $\beta > 0$ is considered. This differential equation appears in the study of mixed convection boundary-layer flows over a vertical surface embedded in a porous medium. In order to prove the existence of solutions satisfying the boundary conditions $f(0) = a \geq 0$, $f'(0) = b \geq 0$ and $f'(+\infty) = 0$ or 1 , we use shooting and consider the initial value problem consisting of the differential equation and the initial conditions $f(0) = a$, $f'(0) = b$ and $f''(0) = c$. For $0 < \beta \leq 1$, we prove that there exists a unique solution such that $f'(+\infty) = 0$, and infinitely many solutions such that $f'(+\infty) = 1$. For $\beta > 1$, we give only partial results and show some differences with the previous case.

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REFERENCES

- [1] R. C. ACKERBERG, *Boundary layer separation at a free stream-line*, J. Fluid Mech., **44** (1970) 211–226.
- [2] M. AÏBOUDI AND B. BRIGHI, *On the solutions of a boundary value problem arising in free convection with prescribed heat flux*, Arch. Math., **93**, 2 (2009) 165–174.
- [3] E. H. ALY, L. ELLIOTT AND D. B. INGHAM, *Mixed convection boundary-layer flows over a vertical surface embedded in a porous medium*, European J. Mech. B/Fluids, **22** (2003) 529–543.
- [4] Z. BELHACHMI, B. BRIGHI AND K. TAOUS, *On a family of differential equations for boundary layer approximations in porous media*, European J. Appl. Math., **12**, 4 (2001) 513–528.
- [5] I. BENSARI-KHELIL, *Ph'D Thesis*, in preparation.
- [6] H. BLASIUS, *Grenzschichten in Flüssigkeiten mit kleiner Reibung*, Z. Math. Phys., **56** (1908) 1–37.
- [7] B. BRIGHI, *On a similarity boundary layer equation*, Z. Anal. Anwendungen, **21**, 4 (2002) 931–948.
- [8] B. BRIGHI, *Sur un problème aux limites associé à l'équation différentielle $f''' + ff'' + 2f'^2 = 0$* , Ann. Sci. Math. Québec, **33**, 1 (2009) 23–37.
- [9] B. BRIGHI, *The equation $f''' + ff'' + g(f') = 0$ and the associated boundary value problems*, Results. Math., **61**, 3–4 (2012) 355–391.
- [10] B. BRIGHI, A. FRUCHARD AND T. SARI, *On the Blasius problem*, Adv. Differential Equations, **13**, 5–6 (2008), 509–600.
- [11] B. BRIGHI AND J.-D. HOERNEL, *On similarity solutions for boundary layer flows with prescribed heat flux*, Math. Methods Appl. Sci., **28**, 4 (2005) 479–503.
- [12] B. BRIGHI AND J.-D. HOERNEL, *On the concave and convex solutions of mixed convection boundary layer approximation in a porous medium*, Appl. Math. Lett., **19** (2006) 69–74.
- [13] B. BRIGHI AND J.-D. HOERNEL, *On a general similarity boundary layer equation*, Acta Math. Univ. Comenian., **77**, 1 (2008) 9–22.
- [14] B. BRIGHI AND T. SARI, *Blowing-up coordinates for a similarity boundary layer equation*, Discrete Contin. Dyn. Syst. A, **12**, 5 (2005) 929–948.
- [15] B. BRIGHI AND J.-C. TSAI, *Similarity solutions arising from a model in high frequency excitation of liquid metal with an antisymmetric magnetic field*, IMA J. Appl. Math., **77** (2012) 157–195.

- [16] P. CHENG AND W. J. MINKOWYCZ, *Free-convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike*, J. Geophys. Res., **82**, 14 (1977) 2040–2044.
- [17] V. M. FALKNER AND S. W. SKAN, *Solutions of the boundary layer equations*, Phil. Mag., **7**, 12 (1931) 865–896.
- [18] M. GUEDDA, *Similarity solutions of differential equations for boundary layer approximations in porous media*, J. Appl. Math. Phys. (ZAMP), **56** (2005) 749–762.
- [19] M. GUEDDA, *Multiple solutions of mixed convection boundary-layer approximations in a porous medium*, Appl. Math. Lett., **19** (2006) 63–68.
- [20] P. HARTMAN, *Ordinary Differential Equations*, Wiley, New-York, 1964.
- [21] H. C. KANG, J. C. YANG AND G. C. YANG, *Existence and uniqueness of concave and convex solutions of mixed convection equation*, Nonlinear Analysis Forum, **13**, 2 (2008) 157–165.
- [22] J. B. MCLEOD, *The existence and uniqueness of a similarity solution arising from separation at a free stream line*, Quart. J. Math. Oxford Ser. (2), **23** (1972) 63–77.
- [23] J. E. PAULLET, *An uncountable number of solutions for a BVP governing Marangoni convection*, Math. Comput. Modelling, **52** (2010) 1708–1715.
- [24] B. B. SINGH AND I. M. CHANDARKI, *On the asymptotic behaviours of solutions of third order non-linear differential equation governing the MHD flow*, Differ. Equ. Appl., **3**, 3 (2011) 385–397.
- [25] J.-C. TSAI AND C.-A. WANG, *A note on similarity solutions for boundary layer flows with prescribed heat flux*, Math. Methods Appl. Sci., **30**, 12 (2007) 1453–1466.
- [26] G. C. YANG, *An extension result of the opposing mixed convection problem arising in boundary layer theory*, Appl. Math. Lett., **38** (2014) 180–185.
- [27] G. C. YANG, L. ZHANG AND L. F. DANG, *Existence and nonexistence of solutions on opposing mixed convection problems in boundary layer theory*, European J. Mech. B/Fluids, **43** (2014) 148–153.