

SIMILARITY SOLUTIONS OF MIXED CONVECTION BOUNDARY-LAYER FLOWS IN A POROUS MEDIUM

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Abstract. The similarity differential equation $f''' + ff'' + \beta f'(f' - 1) = 0$ with $\beta > 0$ is considered. This differential equation appears in the study of mixed convection boundary-layer flows over a vertical surface embedded in a porous medium. In order to prove the existence of solutions satisfying the boundary conditions $f(0) = a \geq 0$, $f'(0) = b \geq 0$ and $f'(+\infty) = 0$ or 1, we use shooting and consider the initial value problem consisting of the differential equation and the initial conditions $f(0) = a$, $f'(0) = b$ and $f''(0) = c$. For $0 < \beta \leq 1$, we prove that there exists a unique solution such that $f'(+\infty) = 0$, and infinitely many solutions such that $f'(+\infty) = 1$. For $\beta > 1$, we give only partial results and show some differences with the previous case.

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