

## NONLINEAR BOUNDARY VALUE PROBLEMS FOR IMPULSIVE DIFFERENTIAL EQUATIONS WITH CAUSAL OPERATORS

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*Abstract.* In this work, we investigate nonlinear boundary value problems for impulsive differential equations with causal operators. Our boundary condition is given by a nonlinear function, and more general than ones given before. To begin with, we prove a comparison theorem. Then by using this theorem, we show the existence of solutions for linear problems. Finally, by using the monotone iterative technique, we obtain the existence of extremal solutions for nonlinear boundary value problems with causal operators. An example satisfying the assumptions is presented.

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