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A PROBLEM INVOLVING THE p–LAPLACIAN OPERATOR

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Abstract. Using a variational technique we guarantee the existence of a solution to the *resonant Lane-Emden* problem $-\Delta_p u = \lambda |u|^{q-2}u$, $u|_{\partial\Omega} = 0$ if and only if a solution to $-\Delta_p u = \lambda |u|^{q-2}u + f$, $u|_{\partial\Omega} = 0$, $f \in L^{p'}(\Omega)$ (p' being the conjugate of p), exists for $q \in (p, p^*)$ under certain condition on λ, where p^* is the Sobolev conjugate of p.

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