

INFINITELY MANY PERIODIC SOLUTIONS TO A CLASS OF PERTURBED SECOND-ORDER IMPULSIVE HAMILTONIAN SYSTEMS

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Abstract. We investigate the existence of infinitely many periodic solutions to a class of perturbed second-order impulsive Hamiltonian systems. Our approach is based on variational methods and critical point theory.

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