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ANTI–PERIODIC SOLUTIONS OF ABEL DIFFERENTIAL EQUATIONS WITH STATE DEPENDENT DISCONTINUITIES

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Abstract. Given $T > 0$, the Abel-like equation $\theta' = f_0 + \sum_{j\in\mathbb{N}} f_j\theta^j$ is generalized to the case where θ and θ' are real functions on $[0,T]$ subject to given state dependent discontinuities. Each f_j is a real function of bounded variation for which $f_j(0) = (-1)^{j+1}f_j(T)$. Under appropriate conditions, this equation is shown to admit a solution of bounded variation on $[0,T]$ which is T-anti-periodic in the sense that $\theta(0) = -\theta(T)$. The contraction principle yields a bound for the rate of uniform convergence to the solution of a sequence of iterates.

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