Differential
Equations
& Applications
Volume 9, Number 2 (2017), 265–283
doi:10.7153/dea-09-21

NON HOMOGENEOUS DIRICHLET PROBLEM FOR THE KDVB EQUATION ON A SEGMENT

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Abstract. We study the Non homogeneous Dirichlet problem with large initial data for the KdVB equation on the interval $x \in (0,1)$

$$\begin{cases} u_t + u_x u - u_{xx} + u_{xxx} = 0,\ t > 0,\ x \in (0,1) \\ u(x,0) = u_0(x),\ x \in (0,1) \\ u(0,t) = u(1,t) = 0,\ t > 0 \\ u_x(1,t) = h(t),\ t > 0. \end{cases} \tag{1}$$

We prove that if the initial data $u_0 \in \mathbf{L}^2$ and boundary data $h(t) \in \mathbf{H}^1_\infty(0,\infty)$ then there exist a unique solution $u \in \mathbf{C}\left([0,\infty);\mathbf{L}^2\right) \cup \mathbf{C}\left((0,\infty);\mathbf{H}^1\right)$ of the initial-boundary value problem (1). We also obtain the large time asymptotic of solution uniformly with respect to $x \in (0,1)$ as $t \to \infty$.

Mathematics subject classification (2010): 35Q35.
Keywords and phrases: dissipative dispersive nonlinear equation, large time asymptotics, KdVB equation.

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