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VIRIAL IDENTITIES FOR NONLINEAR SCHRÖDINGER EQUATIONS WITH A CRITICAL COEFFICIENT INVERSE–SQUARE POTENTIAL

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Abstract. Virial identities for nonlinear Schrödinger equations with some strongly singular potential $(a|x|^{-2})$ are established. Here if $a = a(N) := -(N-2)^2/4$, then $P_{a(N)} := -\Delta + a(N)|x|^{-2}$ is nonnegative selfadjoint in the sense of Friedrichs extension. But the energy class $D((1+P_{a(N)})^{1/2})$ does not coincide with $H^1(\mathbb{R}^N)$. Thus justification of the virial identities has a lot of difficulties. The identities can be applicable for showing blow-up in finite time and for proving the existence of scattering states.

Mathematics subject classification (2010): 35Q55, 35Q40, 81Q15.
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