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PRACTICAL STABILITY OF DIFFERENTIAL EQUATIONS WITH NON–INSTANTANEOUS IMPULSES

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Abstract. The concept of practical stability is generalized to nonlinear differential equations with non-instantaneous impulses. These type of impulses start their action abruptly at some points and then continue on given finite intervals. The practical stability and strict practical stability is studied using Lyapunov like functions and comparison results for scalar differential equations with non-instantaneous impulses. Several sufficient conditions for various types of practical stability, practical quasi stability and strict practical stability are established. Some examples are included to illustrate our theoretical results.

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