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EXISTENCE OF SOLUTION FOR FUNCTIONAL COUPLED SYSTEMS WITH FULL NONLINEAR TERMS AND APPLICATIONS TO A COUPLED MASS–SPRING MODEL

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Abstract. In this paper we consider some boundary value problems composed by coupled systems of second order differential equations with full nonlinearities and general functional boundary conditions verifying some monotone assumptions.

The arguments apply lower and upper solutions method and fixed point theory. Due to an adequate auxiliary problem, including a convenient truncature, there is no need of sign, bound, monotonicity or other growth assumptions on the nonlinearities, besides the Nagumo condition.

An application to a coupled mass-spring system with functional behavior at the final instant is shown.

Mathematics subject classification (2010): 34B15, 34B27, 34L30, 74H20.

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