A COUPLED HYBRID FIXED POINT THEOREM INVOLVING THE SUM OF TWO COUPLED OPERATORS IN A PARTIALLY ORDERED BANACH SPACE WITH APPLICATIONS

BAPURAO C. DHAGE

Abstract. In this paper we prove a coupled hybrid fixed point theorem involving the sum of two coupled operators in a partially ordered Banach space on the lines of Dhage [Math. Student **61** (1992), 81-88] which improve a coupled hybrid fixed point theorem of Dhage [J. Fixed Point Theory Appl. **19** (2017), 3231–3264] under a little stronger condition and correct and improve the hybrid fixed point theorems of Yang *et. al* [J. Fixed Point Theory Appl. **19** (2017), 1661–1678] involving the sum of two operators under weaker conditions. We apply our main abstract coupled hybrid fixed point result to a nonlinear first order coupled linearly perturbed hybrid differential equations with the periodic boundary conditions for proving the existence and approximation of solutions under certain mixed hybrid conditions. The abstract existence result of the coupled periodic boundary value problems is also illustrated by furnishing a numerical example.

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