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MULTIPLE POSITIVE SOLUTIONS FOR A CHOQUARD EQUATION INVOLVING BOTH CONCAVE–CONVEX AND HARDY–LITTLEWOOD–SOBOLEV CRITICAL EXPONENT

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Abstract. In this paper, we consider a Choquard equation involving both concave-convex and Hardy-Littlewood-Sobolev critical exponent. By using the $\mathcal{N}$ehari manifold, fibering maps and the Lusternik-Schnirelman category, we prove that the problem has at least $\operatorname{cat}(\Omega)+1$ distinct positive solutions.

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REFERENCES

[1] CLAUDIANOR O. ALVES, FASHUN GAO, MARCO SQUASSINA, AND MINBO YANG, *Singularly perturbed critical choquard equations*, Journal of Differential Equations **263** (7): 3943–3988, 2017.
[2] CLAUDIANOR O. ALVES AND MINBO YANG, *Multiplicity and concentration of solutions for a quasilinear choquard equation*, Journal of Mathematical Physics **55** (6): 061502, 2014.
[3] C. O. ALVES AND Y. H. DING, *Multiplicity of positive solutions to a p-laplacian equation involving critical nonlinearity*, Journal of Mathematical Analysis and Applications **279** (2): 508–521, 2003.
[4] ANTONIO AMBROSETTI, HAÏM BREZIS, AND GIOVANNA CERAMI, *Combined effects of concave and convex nonlinearities in some elliptic problems*, Journal of Functional Analysis **122** (2): 519–543, 1994.
[5] VIERI BENCI AND GIOVANNA CERAMI, *The effect of the domain topology on the number of positive solutions of nonlinear elliptic problems*, Archive for Rational Mechanics and Analysis **114** (1): 79–93, 1991.
[6] SAMIRA BENMOULOUD, RACHID ECHARGHAOUI, AND SI MOHAMMED SBAÏ, *Multiplicity of positive solutions for a critical quasilinear elliptic system with concave and convex nonlinearities*, Journal of Mathematical Analysis and Applications **396** (1): 375–385, 2012.
[7] HAÏM BRÉZIS AND ELLIOTT LIEB, *A relation between pointwise convergence of functions and convergence of functionals*, Proceedings of the American Mathematical Society **88** (3): 486–490, 1983.
[8] FASHUN GAO AND MINBO YANG, *On nonlocal choquard equations with Hardy-Littlewood-Sobolev criticalexponents*, Journal of Mathematical Analysis and Applications **448** (2): 1006–1041, 2017.
[9] FASHUN GAO AND MINBO YANG, *On the Brezis-Nirenberg type critical problem for nonlinear choquard equation*, Sci China Math, doi:10.1007(/s):10.1007/s11425-016-9067-5, 2017.
[10] TSING-SAN HSU, *Multiple positive solutions for a critical quasilinear elliptic system with concave-convex nonlinearities*, Nonlinear Analysis: Theory, Methods and Applications **71** (7): 2688–2698, 2009.
[11] QIN LI AND ZUODONG YANG, *Multiplicity of positive solutions for a $p-q$-Laplacian system with concaveand critical nonlinearities*, Journal of Mathematical Analysis and Applications **423** (1): 660–680, 2015.
[12] ELLIOTT H. LIEB AND MICHAEL LOSS, *Analysis, volume 14 of graduate studies in mathematics*, American Mathematical Society, Providence, RI, 4, 2001.

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[13] ELLIOTT H. LIEB AND BARRY SIMON, *The hartree-fock theory for coulomb systems*, Communications in Mathematical Physics **53** (3): 185–194, 1977.
[14] VITALY MOROZ AND JEAN VAN SCHAFTINGEN, *A guide to the choquard equation*, preprint arXiv:1606.02158, 2016.
[15] VITALY MOROZ AND JEAN VAN SCHAFTINGEN, *A guide to the choquard equation*, Journal of Fixed Point Theory and Applications, **19** (1): 773–813, 2017.
[16] SOLOMON ISAAKOVICH PEKAR, *Untersuchungen uber die Elektronentheorie der Kristalle*, Akademie-Verlag, 1954.
[17] GABRIELLA TARANTELLO, *On nonhomogeneous elliptic equations involving critical sobolev exponent*, In Annales de l'Institut Henri Poincare (C) Non Linear Analysis, vol. **9**, pp. 281–304, Elsevier, 2016.
[18] M. WILLEM, *Minimax theorems, progress in nonlinear differential equations and applications 24*, 1996.